Source: *Proceedings of the 19th International Modal Analysis Conference (IMAC XIX)*, Orlando, FL, USA, February 5-9, 2001.

FORMULATION OF A MARKET-BASED APPROACH FOR STRUCTURAL CONTROL

Jerome Peter Lynch and **Kincho H. Law**
The John A. Blume Earthquake Engineering Center
Department of Civil and Environmental Engineering
Stanford University, Stanford, CA 94305

ABSTRACT

When designing a control system for structural control, the Linear Quadratic Regulation (LQR) approach is usually taken. While this centralized control approach optimally controls structural deflections during earthquakes, it is not well suited for systems employing a large number of sensors and actuators. As an alternative to LQR, a market-based control system is proposed. Market-based control is a decentralized solution that models the control system around a scarce commodity such as power. The laws of supply and demand are employed to determine the semi-optimal control solution. In both a one and multiple-degree-of-freedom system, market-based control is shown to yield structural response reductions comparable to those obtained from the LQR solution.

NOMENCLATURE

{x}	state-space response of the structure
[F]	state-space system matrix
[G],[L]	location matrix of actuators and loading
{d},{v}	displacement and velocity vectors
J	LQR cost function
{y}	regulated response vector
[R]	cost function weighting matrix
[P]	Riccati matrix
[K], [K_D]	continuous and discrete control gain matrix
P	amount of power bought or sold
p	transaction price of power
K, α	market-base control weighting terms
β	inverse of the supply function slope
ρ	supplier profit
W_i	device wealth
{u}	control force vector
T,Q,R,S	market-based control weighting terms

1. INTRODUCTION

Earthquakes represent one of the most potent naturally occurring load for structures. While many options are available for designing structures to withstand moderate to large earthquakes, the nascent field of structural control has attracted a significant amount of interest by researchers and practitioners alike. Early efforts in structural control focused primarily upon active systems. In an active control system, large actuators are employed to limit structural deflections by applying forces to a structure directly. In 1989, the Kyobashi Seiwa Building in Tokyo, Japan was constructed using an active mass damper making it the first building in the world to use active structural control [1]. While the Kyobashi Seiwa building represents a major achievement for the structural engineering community, it has revealed that active-type structural control systems suffer from some technological and economic limitations.

In response to these inherent limitations, a new semi-active control paradigm has emerged. In this new type of control system, forces are not applied directly, but rather indirectly, to the structure since control devices are used to only change the stiffness or damping properties of the structure. With small energy consumption characteristics, compact sizes and greater degree of reliability, semi-active control devices represent a cost effective solution for limiting structural deflections during large earthquakes [2]. Furthermore, because semi-active control devices do not input energy directly into the system, the devices do not have the ability to destabilize the structure as can be shown using bounded-input bounded-output (BIBO) system theory.

One popular type of semi-active control device is the semi-active variable damper. Typically, variable dampers are located within a lateral resisting frame of a structure through placement at the apex of K-braces. The defining characteristic of the variable damper is that its damping coefficient can be changed during an earthquake excitation thereby indirectly introducing control forces into the system. At least two types of variable dampers have emerged in recent years that employ different types of mechanisms to change their damping coefficients. There is the Semi-Active Hydraulic Damper (SHD) designed by Kajima Corporation, which changes its damping coefficient by varying the orifice opening between adjacent chambers of a hydraulic damper. Such a device can deliver a maximum damping force of 1000 kN using 70 watts of power. Eight SHD semi-active devices have recently been installed in the five-story Kajima-Shizuoka Building in Shizuoka, Japan [3].

The second type of variable damper is the magnetorheological damper being developed at Notre Dame University. This damper changes its coefficient of damping when a magnetic field around the damper's piston causes a change in the viscosity of internal hydraulic fluid. This variable damper can deliver a maximum damping force of 200 kN using only 20 to 50 watts of power [4].

The evolutionary trend of semi-active devices suggest that in time, the shape factor of the devices will become significantly smaller, their capital cost will be reduced and their energy consumption characteristics will be improved. Engineers will have the opportunity to deploy large quantities of semi-active devices throughout a single structure. However, the result is a large-scale control problem entailing hundreds of control devices and sensors.

The large-scale control problem presents new research challenges. Presently, a centralized control system such as Linear Quadratic Regulation (LQR) is used. In such a system, accelerometers measuring structural responses communicate their readings to a central controller who would then communicate control commands to system actuators. The centralized control system will no longer be a prudent design decision for numerous reasons. First, communication between a central controller and a large number of sensors and actuators will require more expensive and faster computers. Furthermore, a single controller represents a single point of failure that can potentially render the control system ineffective if it goes offline. As an alternative, decentralized control methods could prove valuable for the control of structures that employ large numbers of control devices [5]. In the decentralized control approach, a central controller will no longer be used to regulate the structure during an excitation since each semi-active device will have on-board computational means for formulating a semi-optimized control solution. By decentralizing the control solution, the control algorithm can become less reliant on derived models of the structure. This should lead to robust control during the instances of device or structural failure.

2. LINEAR QUADRATIC REGULATION THEORY

For structural control design, a structure is often idealized as a lumped mass shear model. In such a model, each rigid floor represents a degree of freedom of the system that can translate laterally. The state-space representation of the dynamic response of a controlled structure to an external loading can be written as

$$\{\dot{x}(t)\} = [F]\{x(t)\} + [G]\{u(t)\} + [L]\{f(t)\} \qquad (1)$$

where *x(t)* represents the state of the system. This state vector is generally a column vector of system displacements and velocities. The matrix *[F]* is termed the system matrix while *[G]* and *[L]* are the actuator and excitation location matrices, respectively. The system matrix, *[F]*, encapsulates the uncontrolled dynamic response characteristics of the system. By transforming the system matrix from the time domain to the complex domain, a graphical representation of the system's complex roots can be obtained. Plotting these roots, or rather "poles" of the system, on the complex plane, the modal properties such as natural frequency, ω_n, and ratio of critical damping, ξ, can be graphically observed. Furthermore, the complex plane serves as a guide for the system's stability since the left hand side of the complex plane would represent a stable system while poles on the right hand side would imply an unstable system.

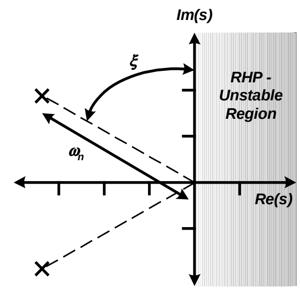

Figure 1 – System Poles on the Complex Plane

The intent behind introducing control to the structure via the *[G]{u(t)}* term of Equation (1), is to initiate migration of the system poles to desired locations in the left half plane on the system. While numerous techniques exist in the control literature for pole placement, one widely used technique is LQR because it is an optimal control technique that weighs the improved response of the structure to the actuation effort required to attain it. In LQR, a cost function, *J*, weighing system response to actuation input is minimized to find the optimal control solution.

$$J = \int_0^{t_f} \left(\{y(\tau)\}^T \{y(\tau)\} + \{u(\tau)\}^T [R] \{u(\tau)\} \right) d\tau \qquad (2)$$

Any system response of interest can be regulated (driven to zero) using LQR by representing this response variable as *{y(t)}*. The relative weighting between the response vector, *{y(t)}*, and actuator effort, *{u(t)}*, is represented by the positive definite matrix *[R].* The criterion of positive definiteness is necessary to ensure that a surface representing the cost function, *J*, is upward convex with a global minimum point defined [6].

The minimum of the cost function is found by adjoining the cost function, *J*, and the constraining equation of motion of the system (Equation 1) with a time dependent Lagrange multiplier, $\lambda(t)$. Neglecting the loading imposed upon the system and assuming the multiplier is proportional to the state vector by the Riccati matrix, *[P],*

$$\lambda(t) = [P]\{x(t)\} \qquad (3)$$

an algebraic solution exists for the Riccati equation that gives the optimal control solution, *{u(t)}*. The solution is proportional to the state of the system by matrix *[K]*.

$$\{u(t)\} = -\frac{1}{2}[R]^{-1}[G]^T[P]\{x(t)\} = -[K]\{x(t)\} \quad (4)$$

The new pole locations of the system employing the LQR controller are the eigenvalues of the augmented state matrix, *[F'] = [F-GK].* The pole locations of the final closed-loop system are dependent upon the form of the cost function, *J*. The variable used to regulate, *{y(t)},* usually influences the pole migration trajectory while the weighting matrix, *[R]* influences how far the poles will migrate along their trajectory paths. For example, let us consider a single-degree-of-freedom system whose system poles are a conjugate pair in the left half side of the complex plane. If the variable for regulation is simply displacement, *{y(t)}* = *{d(t)}*, the final form of the control solution *[K]* will have a dominant non-zero term for the displacement portion of *{x(t)}* and a relatively small non-zero term for the velocity, *{v(t)},* portion. This result is equivalent to an increase in the system stiffness and suggests a pole migration trajectory in the complex plane depicted in Figure 2(a). The final location of poles is dependent upon the value of *[R]* selected. If the weighting term is near infinity, the relative cost of actuation is so high that no control is used in the optimal solution. As *[R]* decreases, more and more control is used. If *[R]* is zero, the final system poles would migrate to infinity.

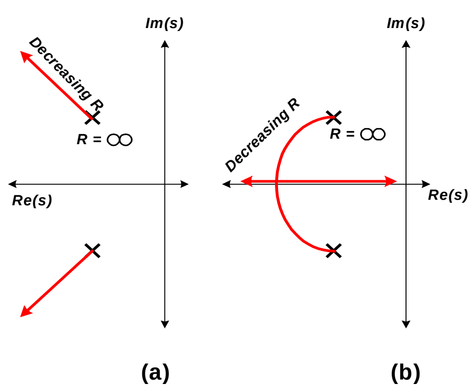

Figure 2 – Effects of Weighting [R] on Locus of System Roots – (a) *{y(t)} = {d(t)}* (b) *{y(t)} = {v(t)}*

Likewise, if the variable of regulation is the velocity response of the system, the gain matrix, *[K],* would have a small term associated with *{d(t)}* and a large one associated with *{v(t)}.* This is equivalent to increased damping in the system as can be seen in Figure 2(b). A combination of *{d(t)}* and *{v(t)}* in the regulation variable would result in a pole migration pattern that would be a combination of the two patterns depicted in Figure 2*.*

3. SEMI-ACTIVE STRUCTURAL CONTROL

For the remaining portion of this paper, our discussion will be limited to structural control systems employing semi-active control devices. In particular, a semi-active control device similar to Kajima's SHD device will be used. Typically, the SHD is installed between the low point of a stiff K-Brace and the floor. Given a command control force, the SHD calculates the damping coefficient by dividing the command force by the relative velocity between the two floors to which the SHD is connected.

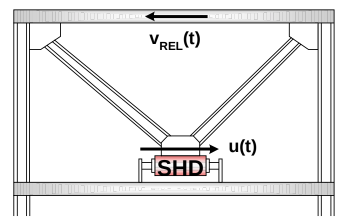

Figure 3 – Installation of a Semi-Active SHD Damper

If the relative velocity between the two floors that the SHD connects has an opposite direction as the desired control force, then the control force is applied by the SHD. If the relative velocity is in the same direction, no control force can be applied and the damper is set to its minimum damping coefficient. Given that the control device is attached to the floor above through a flexible K-brace, the SHD and brace are modeled as one Maxwell damping element in which a dashpot and spring are connected in series. Some of the properties associated with the SHD device are summarized in Table 1 [3].

Maximum Control Force	1000 kN
Maximum Displacement	+/- 6 cm
Stiffness of SHD	400,000 kN/m
Maximum Damping Coefficient	200,000 kN-s/m
Minimum Damping Coefficient	1,000 kN-s/m
Maximum Shaft Velocity	25 cm/s
Weight	1300 kg

Table 1 – SHD Specifications

4. OVERVIEW OF MARKET-BASED CONTROL

In observing free capitalistic markets, it is evident that through the individual intentions of both consumers and sellers alike, an efficient means of societal resource allocation exists. The complex laws of supply and demand, first set forth by Adam Smith in 1776, are the fundamental building blocks in determining the equilibrium price of goods in a decentralized economy. What the market is excellent at "controlling" is the price we pay for goods and the salary we receive for our services. A free market optimally controls the prices of all goods and services. The historically poor performance of centralized economies is evidence of the difficulty associated with centrally controlling a market.

Borrowing the concept of a marketplace for application in a control system is rather new to the field of control. Investigations have been made into the use of market-based control techniques in the field of MEM's (micro-electrical machines) where hundreds of actuators and sensors are employed in a system of high plant uncertainty and potential actuation and sensor failure [7]. One area that market-based control has made significant advances is in the area of computer resource allocation (i.e., memory, network traffic, and processor time) [8].

A structure employing a large number of sensors and actuators can be modeled as a market place that centers around the buying and selling of the scarce resource of power. The cost efficiency of any structural control system is indirectly proportional to the consumption of power the system requires. If a system consumes less power, it represents a more cost efficient solution. Therefore, allowing actuators to enter a market place for purchasing power, the local interaction of the system's actuators can potentially attain an efficient global control system that can adequately control a structure's deflections while at the same time use power in an efficient manner. Techniques like LQR guarantee an optimal solution whereas the market-based method makes no such promises. However, the market-based control solution can be viewed as semi-optimal.

Since there is no central authority imposing rigidity to the system infrastructure, market-based control systems are easily expandable and maintainable. Furthermore, tolerances for model uncertainty would be higher in a market based system than in a classical centrally controlled system. All of these benefits make market-based control an attractive option to be considered for structural control where information about the plant (structure) is not always known with a high degree of certainty or when use of a central controller has its technological and economic limitations.

5. MARKET-BASED CONTROL FOR A ONE-DEGREE-OF-FREEDOM SEMI-ACTIVE CONTROL SYSTEM

Let us consider a simple one-degree-of-freedom structure controlled by an SHD device. Since the system has only one actuator, it does not represent a large-scale economy or marketplace. Nonetheless, the single-degree-of-freedom system is ideal to formulate mathematically the laws of supply and demand.

The scarce commodity of interest in the system is power, P. The SHD devices need power to successfully limit the deflections of the structure. The control force applied to the structure by each damper will be directly proportional to the amount of power purchased by each device. Each actuator will have an opportunity to purchase at the market's equilibrium price, p. When the price of power is high relative to the actuator's overall wealth, it is inclined to purchase less. In the opposite case of inexpensive power, the actuators will purchase more. This verbal explanation of device demand can be graphically represented in many ways (e.g. a straight line, parabola, etc). For simplicity, a simple linear demand function is proposed. The only restriction that is faced when proposing demand functions is that when structural response is high (high displacement and high velocity) it is expected that more power will be demanded. For a linear demand function, the slope and P-axis intercept are sufficient to fully characterize the function. Both properties are proposed to be a function of the structure's displacement and velocity.

$$P_{DEM} = -\left|f(x,\dot{x})\right| p + \left|g(x,\dot{x})\right| \qquad (5)$$

When the structure's displacement and velocity grow, it is natural to expect the demand for power to grow proportionally. To represent the growth in demand, it is proposed that the intercept and slope of the demand function take on the following form.

$$f(x,\dot{x}) = \frac{1}{x+\alpha\dot{x}} \qquad (6)$$
$$g(x,\dot{x}) = K(x+\alpha\dot{x})$$

Figure 4 depicts how the selection of the demand function's intercept and slope influence the growth of demand for large structural responses.

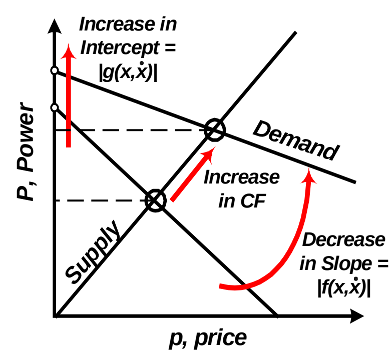

Figure 4 - Growth of Demand and Control Force to Large System Responses

For the law of supply, each producer of power (i.e., semi-active device battery) seeks to maximize profits associated with selling power to the system buyers. The profit, ρ, obtained by the producer is proportional to the power sold to the device, P, and is a function of the cost of power production, C, which is assumed to be proportional to the square of the power produced: $\rho = pP - C = pP - \beta P^2$. Profit is maximized with respect to power, resulting in a linear supply function.

$$P = \frac{1}{\beta} p \qquad (7)$$

The equilibrium price of power at each time step is determined by finding the point where supply equals demand. For the selected supply and demand functions, the resulting control force can be found.

$$CF = K\left(\frac{\left| x+\alpha \dot{x} \right|}{\left| x+\alpha \dot{x} \right| + \beta} x + \frac{\left| x+\alpha \dot{x} \right|}{\left| x+\alpha \dot{x} \right| + \beta} \alpha \dot{x} \right) \qquad (8)$$

The control solution of the market-based control method is quite similar to the control solution obtained from the LQR approach. In the market-based solution, the coefficients of the displacement and velocity terms vary in response to the system's response. In LQR, the coefficients are static.

$$CF_{LQR} = K\left(x + \alpha \dot{x} \right) \qquad (9)$$

To observe the performance of the proposed market-based control method, an analysis is performed upon a single-degree-of-freedom structure controlled by an SHD device during a large seismic excitation. In an attempt to properly scale a single-degree-of-freedom system to the SHD device, the mass and stiffness of the structure are selected to be 200,000 kg and 20,000 kN/m respectively. This results in a natural period of 0.628 seconds. Viscous damping of the structure is assumed to be roughly 2% of critical damping. The seismic excitation of interest in this analysis is the unscaled record of El Centro, 1940.

The properties of the SHDs used in this analysis are the same as those shown in Table 1. The stiffness of the K-brace is selected to be roughly 15,580 kN/m. While outside the scope of this study, it can been shown that the choice of effective stiffness of the K-brace is important in the performance of the control system relative to the performance of the SHD when set to its maximum damping coefficient. When the ratio of SHD bracing stiffness to lateral structural stiffness, also known as the stiffness rate, is high, the damper operated by the control system will not perform better than when the damper is fixed at its maximum damping value. When the stiffness rate is low, as is typical of SHD installations in high rise structures, the controlled damper will perform better than the damper fixed at its maximum damping coefficient. The value selected for this study is considered a moderately low stiffness rate and therefore it can be expected that the control system will outperform the static damper set to its maximum damping value of 20,000 kN-s/m.

First, the structural response is measured with the structure employing no SHD device, an SHD set at a fixed damping value of 1,000 kN-s/m and an SHD set at its maximum permissible value of 200,000 kN-s/m. The responses of the three runs are shown in Figure 5. Of particular interest is the maximum displacement of the one-degree-of-freedom structure. For the structure employing no semi-active control device, the maximum displacement is 4.3 cm. When one SHD device is installed and set at its minimum damping coefficient, the maximum structural displacement is reduced to 2.8 cm. If the SHD's damping coefficient is set at its maximum damping coefficient, displacement can be further reduced to 2.4 cm.

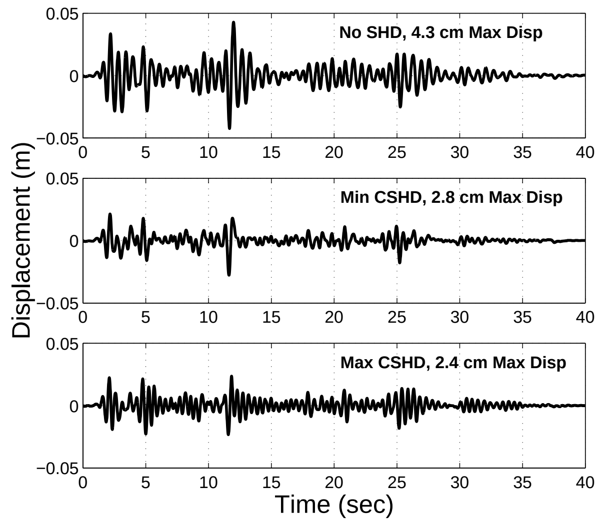

Figure 5 – Single DOF System Response – No SHD Installed, Minimum Damping, Maximum Damping

The LQR control solution is determined after selecting the appropriate *[Q]* and *[R]* weighting matrices. One of the primary goals of the control solution is to minimize structural deflections in order to keep structural displacements within an elastic limit. To reflect this goal, the diagonal element of [Q] associated with displacement is selected to be significantly larger than the element associated with velocity.

$$[Q] = \begin{bmatrix} 10{,}000 & 0 \\ 0 & 1 \end{bmatrix} \qquad R = 1x10^{-13}$$

The resulting discrete LQR gain is found to be $[K_D] = [-2.2x10^{8} \;\; -9.7x10^{6}]$. With the LQR solution commanding the SHD, the structural response of the single-degree-of-freedom structure is reduced to a maximum displacement of 2.17 cm.

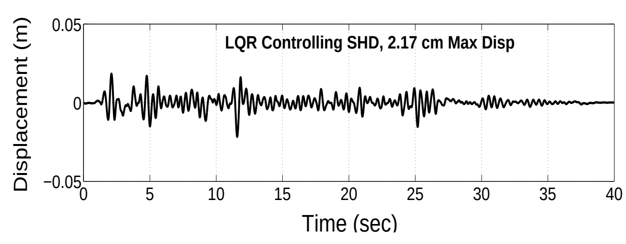

Figure 6 – Single DOF System Response - LQR Control Solution

The market-based control method is also implemented on the system. The tuning parameters associated with the demand and supply functions, *K*, α, and β, need to be judiciously selected to obtain control results competitive with LQR. After a rigorous search for near optimum values of the parameters, the following are obtained: $K = -2190$, $\alpha = 4.42 \times 10^{-4}$, and $\beta = 1\times10^{-6}$.

The results of the structure's response to the El Centro record under market-based control is nearly the same as that obtained in the LQR method. For the market-based method, the maximum story displacement is 2.20 cm compared to LQR's 2.17 cm as seen in Figure 7. This result is expected due to the similarities between the final form of the control force equations for both the market-based and LQR controllers.

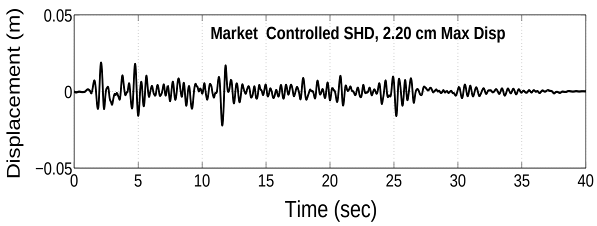

Figure 7 – Single DOF System Response - Market-Based Control Solution

As previously discussed, the location of system poles can give incredible insight to the system response properties. For the LQR solution, the system poles move from their uncontrolled location to a fixed location of greater natural frequency, ω_n and damping ratio, ξ. In contrast, for the market-based method, the form of the equation of the control force suggests that the poles have a tendency to migrate over time around some fixed point as shown in Figure 8.

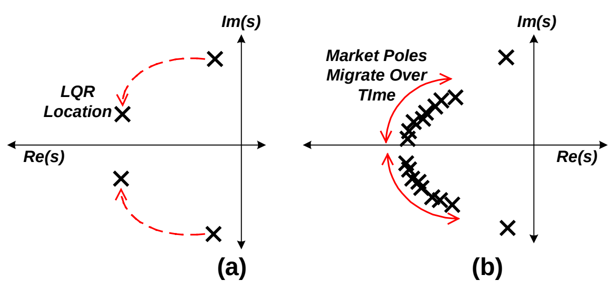

Figure 8 – Pole Locations of (a) LQR and (b) Market

6. MARKET-BASED CONTROL FOR A MULTI-DEGREE-OF-FREEDOM SEMI-ACTIVE CONTROL SYSTEM

While the results from the market-based controller of the one-degree-of-freedom system are nearly equal to those of the LQR controller, the true test of the method comes from its application in a multiple-degree-of-freedom system. In such a system, the meaning of a market has significantly more substance than in its one-degree-of-freedom counterpart. The system's multiple actuators will form the marketplace's buying population while the power distribution system will form the sellers. Each actuator will be provided with an amount of fictitious wealth, W_i, with which it can buy power at the market price. Actuator wealth, not present in the original single-degree-of-freedom system formulation, will have some influence on the demand function. With more relative wealth, a buyer is inclined to buy more power.

$$P_i = \left(\frac{-p}{\left| Tx_i + Q\dot{x}_i \right|} + \left| Rx_i + S\dot{x}_i \right| \right) W_i \qquad (10)$$

The equilibrium price will be determined by finding the point where the function representing the summation of the individual demands of the actuators equals the summation of the seller's supply functions (total of n sellers in the system).

$$p_{eq} = \frac{\sum_{i=1}^{n} W_i \left| Rx_i + S\dot{x}_i \right|}{\dfrac{n}{\beta} + \sum_{i=1}^{n} \dfrac{W_i}{\left| Tx_i + Q\dot{x}_i \right|}} \qquad (11)$$

Once the market price is established, each actuator will only buy power if the market price at that time step does not exceed its wealth. Once all actuators have purchased power, the money is subtracted from each buyer's wealth and distributed evenly through out the system to each actuator, regardless of whether the actuator purchased power at that step or not. Similar to the previous single-degree-of-freedom system, the control force is proportional to the power purchased.

Kajima has completed construction of the Kajima Shizuoka Building, the first building in the world to employ a semi-active variable damper control system for large earthquakes [3]. The Shizuoka building is a five-story steel frame structure that has two different SHD devices upon each of its first four floors. The different SHD dampers are connected to the lateral steel frame through a steel K-brace located on the two exterior sides of the structure. Taking the published properties of the Shizuoka structure, a lumped mass shear model was made for analysis as shown in Figure 9. In Figure 9, all stiffness properties shown are representative of the summation of stiffness per story.

For this particular system, three different earthquake excitations are considered. In particular the El Centro (1940), Loma Prieta (Stanford-SLAC), and Northridge (Canoga Park) ground motion records are selected with each record scaled to a maximum peak ground velocity of 60 cm/s. This is necessary to roughly ensure some uniformity in the maximum input energy to each structure.

Again, the market-based control solution is implemented and compared to both the uncontrolled structural response and response of the system with the SHDs set at their minimum damping coefficients. For the multi-degree-of-freedom system, maximum story drifts are used as the response characteristic of comparison since this is a good indicator of the structural elastic displacements.

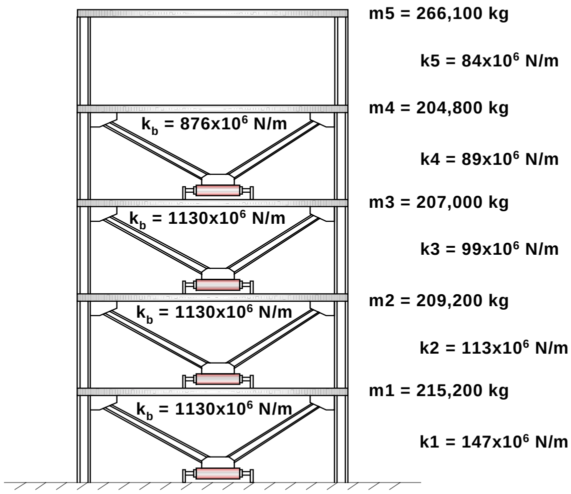

Figure 9 – Model of the Kajima-Shizuoka Building

As can be seen by the results presented in Figure 10, the market-based method is successful in reducing story displacements by more than 50%. Due to the absence of an SHD device on the fifth story of the structure, this story's deflection does not experience any reduction in its drift.

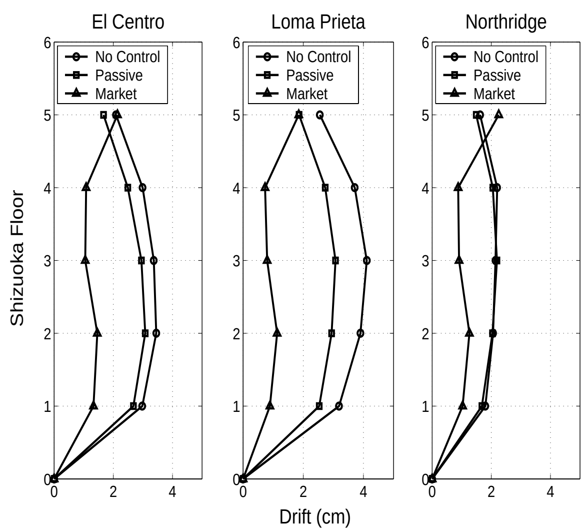

Figure 10 – Story Drift Response to Market Based Control

7. CONCLUSION

The market-based control method has been illustrated to be an effective control technique for limiting structural deflections during earthquakes. For the one-degree-of-freedom system, the market-based method yields results nearly identical to the LQR results. For the Shizuoka multi-degree-of-freedom, significant story drift reductions were attained with market-based control.

While these results represent a significant advancement of the market-based control method, more work is needed to render it a self-reliant decentralized control solution. One area of future investigation is how to allow the system to tune the current weighting parameters of the demand function (*R,S,T,Q*) which would make the approach model independent and adaptable over time. Furthermore, a means of inter-device communication must be developed to eliminate the need of the central controller; for example, using a hierarchical data aggregation model.

ACKNOWLEDGEMENTS

This research is partially sponsored by NSF, Grant No. CMS-9988909. We gratefully acknowledge the fruitful suggestions received from Dr. Andrew Berlin and Dr. Warren Jackson of Xerox PARC, Narito Kurata, Tomohiko Hatada, and Akihiro Kondo of the Kobori Research Institute of Kajima Corporation, and Prof. Stephen Rock of Stanford University.

REFERENCES

[1] Kobori, T., Koshika, N., Yamada, K., Ikeda, Y., "Seismic-Response-Controlled Structure with Active Mass Driver System. Part 1:Design." *Earthquake Engineering and Structural Dynamics.* Vol 20, pp 133-149 (1991).

[2] Soong, T. T. and Spencer Jr., B. F., "Active, Semi-Active and Hybrid Control of Structures." *Proceedings of the 12th World Conference on Earthquake Engineering*. New Zealand (2000).

[3] Kurata, N., Kobori, T., Takahashi, M., Niwa, N. Midorikawa, H., "Actual Seismic Response Controlled Building with Semi-Active Damper System." *Earthquake Engineering and Structural Dynamics*. Vol 28, pp 1427-1447 (1999).

[4] Spencer Jr., B., Yang, G., Carlson, J.D., Sain, M.K., "Smart Dampers for Seismic Protection of Structures: A Full-Scale Study". *Proceedings of the 2nd World Conference on Structural Control.* pp 417-426. Kyoto, Japan (1998).

[5] Lunze, J., *Feedback Control of Large-Scale Systems*. pp 1-6, Prentice Hall, New York (1992).

[6] Stengel, R., *Optimal Control and Estimation.* pp 184 - 210, Dover, New York (1994).

[7] Guenther, O., Hogg, T., Huberman, B.A., "Control for Unstable Structures" *SPIE Smart Structures and Materials: Mathematics and Control in Smart Structures*. Vol 3039, pp 754-763 (1997).

[8] Clearwater, S., *Market-Based Control: A Paradigm for Distributed Resource Allocation* World Scientific Publishing, Singapore (1996).