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DECENTRALIZED CONTROL TECHNIQUES FOR LARGE-SCALE CIVIL STRUCTURAL SYSTEMS

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ABSTRACT

As the state of structural control technology advances towards smaller, more power efficient and less expensive control devices, future structural control systems are likely to be deployed with large numbers of control devices and sensors. The result is a large-scale control problem characterized by having high sensor and actuator density. Such systems are better suited for decentralized control. Decentralized control techniques decompose the global system to a set of local sub-systems for which local controllers are designed. This paper discusses three decentralized control techniques: modified LQR control, decentralized optimal control, and market-based control. A 20-story analytical structure, employing semi-active control devices, is used as the benchmark structure for comparing the performance and power consumption characteristics of the various control techniques considered.

NOMENCLATURE

{x}	state-space response of the structure
{u}	actuation control forces
{f}	external disturbance vector
{y}	measured system output
[A]	state-space system matrix
[B]	actuator location matrix
[C]	sensor location matrix
[Q]	positive definite state weighting matrix
[R]	positive definite control weighting matrix
[P]	Ricatti matrix
[G]	feedback control gain
J	controller cost function
Φ(t)	state trajectory transformation matrix
P	marketplace power
p	price of power

1. INTRODUCTION

The incorporation of technology in the design of civil structures that enhances the lateral integrity of the structure during natural excitations is not new to the field of structural engineering. To combat the tremendous forces placed upon civil structures during earthquakes and winds, various types of control systems have been designed since the inception of the concept of structural control by Yao in 1972 [1]. Since that time, significant interest in the approach has resulted in the use of various types of control systems differing in performance, cost and power usage specifications. Early efforts concentrated upon the design of active control systems that are characterized by one or two large actuators, such as the active mass damper system, applying forces directly to a structure [2]. However, active-control systems suffer from high costs and limited performance especially during large seismic disturbances.

A new control paradigm, termed semi-active control, has emerged as a viable alternative to active-control. In this new type of control system, forces are not applied directly, but rather indirectly to the structure through the use of system control devices that can change the stiffness and/or damping properties of the overall structural system. With small energy consumption characteristics, compact sizes and greater degree of reliability, semi-active control devices represent a cost effective solution for limiting structural deflections during large earthquakes [3]. Various types of semi-active control devices have been proposed with the semi-active variable damper being the most popular. Using a semi-active variable damper, a control force of nearly 1,000 kN can be generated using less than 100 watts of power [4].

The evolutionary trend of semi-active devices suggest that in time, the shape factor of the devices will continue to become significantly smaller, their capital cost will be reduced and their energy consumption characteristics will be improved. The deployment of large numbers of semi-active control devices will result in a system of high actuator density.

An important component of any control solution is the sensing system. Control systems depend heavily upon sensors to relay real time information regarding the state of the system, to a centralized controller. In practice, only a few sensors are installed at strategic locations in a structure for state feedback because of their high installation and maintenance costs. Significant advances are being made in the structural health monitoring research community towards making low cost and highly reliable sensors for dense senor array deployment [5]. Two enabling technologies are making this possible: microprocessors and wireless communications.

New sensing units are incorporating microprocessors at their core because of their low cost and small size. Some of the responsibilities microprocessors have include overall unit operation, interrogation of the measurement data and communication of the interrogated data. Traditional monitoring systems for structures are based upon a centralized data logging architecture with sensors wired directly to a single logging computer. Such an approach is vulnerable to a single point of failure of the data logger, but also suffers from high installation costs associated with the installation of wires. The incorporation of wireless communication capabilities within a sensing unit allows for wireless communication of all system data to all other sensing nodes in the system without wires. Advanced wireless sensing units have been designed and their viability for application in structural monitoring systems tested [6, 7].

The trend of reducing the size and cost of semi-active control devices and structural response sensors will continue. The result will be an overall increase in the number of actuators and sensors used by engineers, which will increase the complexity of the structural control system. Introducing a dense array of actuators and sensors into an already complex structure results in a control system characterized by high system dimensionality and is often termed a large-scale control problem.

With increases in system dimensionality, computations required by a centralized controller increase faster than at a linear rate [8]. The real time demands of the control system make it necessary to be aware of the number of calculations that can be made in each operation cycle. An additional failure of centrality occurs when considering the spatial separation of system sensors. Cost and reliability of communication links between system sensors and a controller need to be considered [9].

As a result, the division of the control problem into a collective set of smaller sub-systems that can be controlled on a local level by decentralized controllers is a possible alternative to the centralized controller. Control computations can now be performed in parallel using the decentralized distributed control systems. The microprocessor computational core of the sensing units can be exploited for performing these computations of local control forces. This control approach of decomposing the global system to a set of sub-systems is termed decentralized control or distributed control.

2. DECENTRALIZED CONTROL

The information structure of the control system defines the class of the control approach (centralized versus decentralized). In the centralized approach, complete knowledge of the system plant (*a priori* information) and a complete set of state data (*a posteriori* information) is assumed during implementation. In decentralized control, local controllers only have access to a portion of the global information. The amount and type of information available to each sub-system controller defines the non-classical information structure of the decentralized control approach.

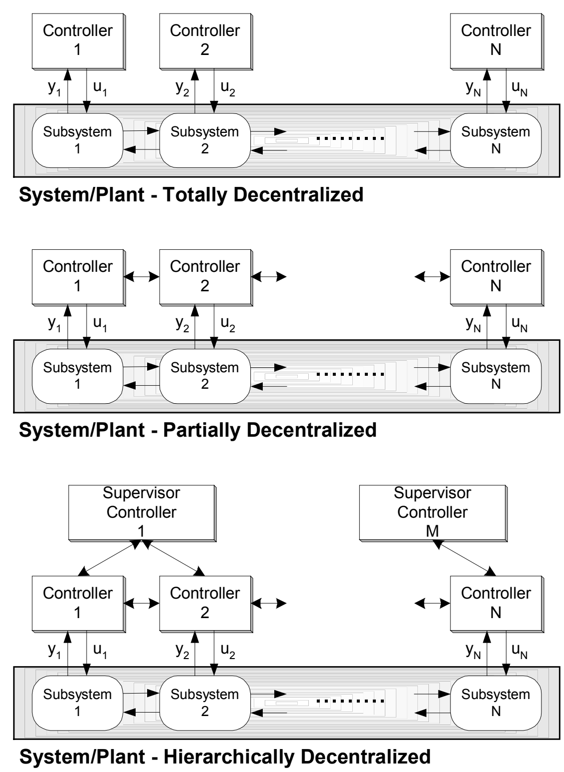

Figure 1 – Decentralized Information Structures

Three types of decentralized information structures can be identified: total, partial and hierarchical decentralized control structures [10]. Figure 1 illustrates the three forms of the decentralized information structure. A totally decentralized control structure provides local *a posteriori* information to each controller with no information exchanged between local controllers. Knowledge of how the control actions of the local controller affect the overall system response is not known. Information transfer between local controllers leads to partially decentralized control. While the amount of information exchange is kept as low as possible, it provides a way of ensuring partial knowledge of how the local controller is affecting the global system. The last type, hierarchical decentralized control, adds an additional layer of vertical information flow to additional controllers situated above that of the local controllers. The principal role of the controllers of the higher level is to ensure concordant behavior between the lower local controllers leading to improved overall global performance.

A dynamic multivariable system can be represented mathematically in state-space form. Many of the widely used classical control approaches depend upon representing the dynamic system as a linear time invariant model shown in Equation 1.

$$\{\dot{x}(t)\} = [A]\{x(t)\} + [B]\{u(t)\} + [F]\{f(t)\}$$
$$\{y(t)\} = [C]\{x(t)\} \quad (1)$$

The state of the system is represented by the vector *{x(t)}* and is often a vector of the displacement and velocity responses at the system's degrees of freedom. The system matrix, *[A]*, encapsulates the uncontrolled response characteristics of the open-loop system. With the application of control to the system, the control forces, *{u(t)}*, are placed upon the system through the location matrix, *[B]*. External dynamic disturbances to the system are represented by *[F]{f(t)}*. The system is monitored by an array of sensors that provide the measurement vector, *{y(t)}*. The measurement vector of the system is related to the state of the system through the sensor location matrix, *[C]*.

The model depicted in Equation 1 facilitates the use of a traditional centralized controller. By modifying Equation 1 according to the structural constraints of the various subsystems, the decentralization of the control solution can be depicted. At the subsystem level, the control force input to the system and the sensors to measure subsystem output are localized as shown in Equation 2. Here, the system is decomposed into *N* subsystems.

$$\{\dot{x}(t)\} = [A]\{x(t)\} + \sum_{i=1}^{N}[B_i]\{u_i(t)\} + [F]\{f(t)\}$$
$$\{y_i(t)\} = [C_i]\{x(t)\} \quad (2)$$

This only represents one convenient way of depicting the decentralization of the problem along the totally or partially decentralized architectures depicted in Figure 1.

Various decentralized control techniques exist but for this paper, the discussion will be limited to only three decentralized control techniques. First, the optimal centralized controller for a structural system will be modified to result in a decentralized but sub-optimal control solution. For a strongly coupled system like a civil structure, this extension represents a logical approach of simply modifying the familiar centralized controller to fit a decentralized architecture. While such an approach is not optimal, the sub-optimal solution might be outweighed by its ease of implementation. The second decentralized approach, termed optimal decentralized control, is formulated beginning with the decentralized architecture of the system and the control problem optimized accordingly. The last control approach is more of a phenomenological approach termed market-based control. The market-based controller is suitable for implementation upon a hierarchically decentralized information structure.

3. BENCHMARK STRUCTURE

A benchmark structure is selected for comparison of the selected distributed control techniques as shown in Figure 2. The benchmark structure is a 20-story steel structure designed for the Structural Engineers Association of California (SAC) project. The building was designed to current seismic codes and represents a realistic building design for the southern California region. In modeling the structure, a linear model is employed with full column, beam and joint deformation as provided by the nonlinear benchmark project at the University of Notre Dame [11]. The natural frequencies of the first five modes of the model are: 0.26, 0.75, 1.3, 1.8, and 2.4 Hz

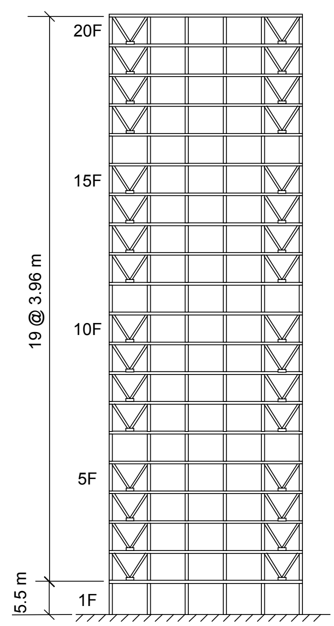

Figure 2 – 20-Story Benchmark Structure

To limit the response of the structure to external disturbances, semi-active control devices are attached to the structure at various locations. Control devices with a 1000 kN capacity are used, connected to the structure between adjacent floors. The control devices are installed in a manner that establishes four distinct subsystems with four control devices in each with no device bridging any two subsystems. In order to generate large control forces, each floor has two control devices installed except the first, sixth, eleventh and sixteenth floors. In total, thirty-two semi-active devices are used in the structural frame.

Three earthquake loadings are applied to the structure for each proposed decentralized control solution. The El Centro N-S record of the Imperial Valley earthquake of 1940, the Hachinohe City N-S record of the Tokachi-oki earthquake of 1968, and the Sylmar Hospital N-S record of the Northridge earthquake of 1994 are the three records of interest. The El Centro and Hachinohe earthquakes, with peak absolute accelerations of 3.42 and 2.25 m/s^2 respectively, represent far-field records while the Northridge record, with a peak absolute acceleration of 8.27 m/s^2, is considered near-field.

4. DECENTRALIZED EXTENSIONS OF THE CENTRALIZED CONTROLLER

The purpose of the *[B]{u(t)}* term of Equation 1 is to control the response of the open-loop system in a manner that is beneficial. Many centralized control approaches can be considered for this task, but often times, the designer seeks an optimal control design. In a control context, optimal implies maximum benefit with minimum work. One widely used technique that maximizes control benefit with a minimum of input power is the Linear Quadratic Regulation (LQR) controller. The derivation of the LQR controller requires full knowledge of the system state at all times and therefore represents a centralized control approach.

In LQR, a cost function, *J*, weighing system response to actuation input is minimized to find the optimal control solution.

$$J = \frac{1}{2}\int_0^{t_f}\left(\{x(\tau)\}^T[Q]\{x(\tau)\} + \{u(\tau)\}^T[R]\{u(\tau)\}\right)d\tau \qquad (3)$$

The state of the system, *{x(t)}*, is regulated (driven to a zero state) by the controller using a minimum amount of control power. The relative weighting between the regulation of the state vector and actuator effort is represented by the positive definite matrices, *[Q]* and *[R]*. The criterion of positive definiteness is necessary to ensure that the cost function surface of *J*, is upward convex with a global minimum point defined [12].

Constrained by the equation of motion (Equation 1), the cost function, *J*, is minimized. An intermediate result of the minimization of the cost function is the Ricatti equation with *[P]* representing the Ricatti matrix:

$$[A]^T[P] + [P][A] - [P][B][R]^{-1}[B]^T[P] + [Q] = 0 \qquad (4)$$

An algebraic solution exists for the Riccati equation that gives the optimal control solution, *{u(t)}*. The solution is proportional to the state of the system by matrix *[G]*.

$$\{u(t)\} = -[R]^{-1}[B]^T[P]\{x(t)\} = -[G]\{x(t)\} \qquad (5)$$

For the 20-story benchmark structure, the gain matrix, *[G]*, is characterized by a strong dominance of the diagonal terms of the matrix. Figure 3, a three dimensional representation of the values of the *[G]* matrix, illustrates the relative dominance of the diagonal terms with the front most diagonal corresponding to displacement and the back most velocity.

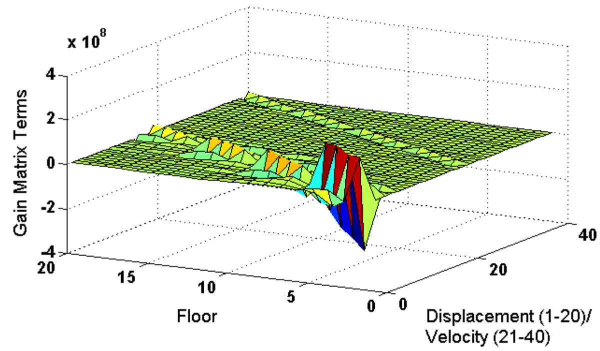

Figure 3 – Terms of the LQR Gain Matrix

The dominance of the diagonal terms is expected because the actuators and sensors are collocated. The form of the gain matrix is convenient and can be exploited with minor modifications to provide a decentralized control solution. All gain terms outside of the four subsystems of the 20-story structure are zeroed, as shown in Figure 4. Such a solution would represent a sub-optimal controller that is a close approximation of the optimal controller. As the dominance of the off-diagonal terms increase, the decentralized extension of the centralized controller diverges further from optimality.

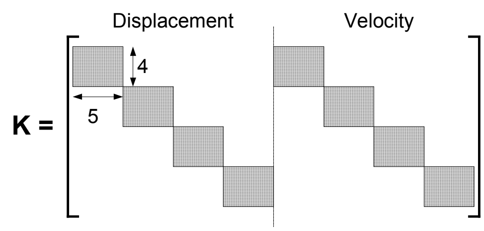

Figure 4 – Non-zero Terms of the New Gain Matrix

Figure 5 shows the response of the 20-story benchmark structure employing the centralized controller and the decentralized extension of the centralized controller for all three earthquakes. In calculating the LQR controller gain, the relative weighting between *[Q]* and *[R]* is 3×10^{14}. As shown in Figure 5, the LQR controller and the decentralized extension represent significant gains in reducing the overall response of the structure. The difference between the centralized LQR and decentralized extension is best illustrated when considering a plot of the normalized inter-story drift response of the structure. The decentralized extension exhibits degraded performance with more erratic differentiation in drifts demand from one story to the next. This is in contrast to the well behaved maximum drifts of the centralized LQR controller. The zero off-diagonal terms of the decentralized extension appear to be the contributing factor in the reduction of the optimality of the control solution.

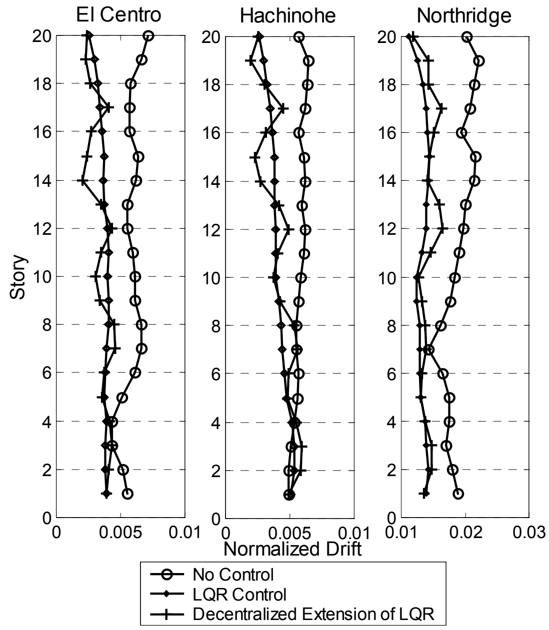

Figure 5 – Inter-story Drift Demands: Centralized LQR versus the Decentralized LQR Extension

5. OPTIMAL DECENTRALIZED CONTROL

The decentralized extension of the centralized LQR controller, while yielding suitable performance, does not represent an optimal solution. A reformulation is necessary to derive an optimal decentralized controller. The derivation will parallel that of the centralized LQR controller.

The cost function used in the derivation of the LQR controller (Equation 3) is now minimized subject to Equation 2. Unlike the minimization of the cost function when constrained by Equation 1, the minimization in this case can yield a non-linear feedback controller [8]. However, to remain consistent with the linear feedback control structure, the feedback control law is explicitly defined with the control gain, *[G']*, which is a block diagonal matrix of the various subsystem displacement and velocity gain matrices, $[G_{Di}]$ *and* $[G_{Vi}]$, as shown in Equation 6.

$$[G'] = \begin{bmatrix} G_{D1} & & & G_{V1} & & \\ & \ddots & & & \ddots & \\ & & G_{DN} & & & G_{VN} \end{bmatrix} \tag{6}$$

The optimization of the cost function is further constrained by Equation 6 to ensure that a linear feedback controller results. A direct result of this constraint is that the optimization is now dependent upon the initial state of the system. The state trajectory, $\{x(t)\}$, is a direct function of the initial state as stated in Equation 7.

$$\{x(t)\} = e^{[A]-[B][G'][C]}\{x_O\} = \Phi(t)\{x_O\} \tag{7}$$

The cost function of is written to reflect the dependence of the state trajectory on its initial state.

$$J = \frac{1}{2}\{x_O\}^T \int_0^{t_f} \Phi(t)^T \left([Q]+[C]^T[G']^T[R][G'][C]\right)\Phi(t)dt\{x_O\} \tag{8}$$

Provided that the weighting matrix, *[Q]*, is symmetric, the cost function of Equation 8 is equivalent to

$$\begin{aligned} J &= \frac{1}{2}\ trace\left([P]\{x_O\}\{x_O\}^T\right) \\ [P] &= \int_0^{t_f} \Phi(t)^T\left([Q]+[C]^T[G']^T[R][G'][C]\right)\Phi(t)dt \end{aligned} \tag{9}$$

Here, *[P]* is the solution to the well known Lyapunov stability equation

$$\begin{aligned} &([A]-[B][G'][C])^T[P]+[P]([A]-[B][G'][C]) \\ &\qquad +[C]^T[G']^T[R][G'][C]+[Q]=0 \end{aligned} \tag{10}$$

Provided that the initial state of the system represents an unknown, the mean value of the minimization of the cost function is sought over all possible $\{x_O\}$. This is equivalent to simply minimizing Equation 11.

$$\tilde{J} = trace[P] \tag{11}$$

With a relative weighting of $1.2\text{x}10^{14}$ between *[Q]* and *[R]*, an optimal decentralized controller is designed following the iterative procedure proposed by Lunze [8]. The resulting controller gain for the benchmark structure is in the appropriate form that is suitable for implementation in the system's four totally decentralized subsystems as shown in Figure 6. One interesting observation is how the optimal decentralized controller compensates for its decentralization with a heavier emphasis upon the feedback of the state's velocity terms when compared to the gain matrix of the centralized controller of Figure 3.

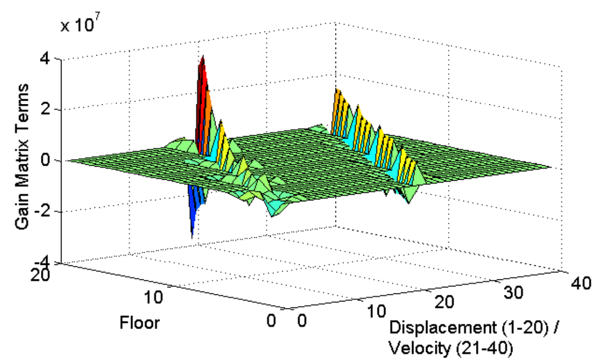

Figure 6 – Terms of the Optimal Decentralized Gain Matrix

As presented in Figure 7, the results of the optimal decentralized controller are compared against those obtained by the centralized optimal controller. The results of the decentralized controller are quite similar to those of the centralized controller with only a slight degradation of performance at the higher levels of the structure. However, the controller still represents an optimal solution subjected to the limitations of *a posteriori* information flow in the system infrastructure. The centralized controller has the advantage of transferring information regarding the state of the system at the structure's lower levels to the actuators at the structure's higher level giving the centralized controller better performance at the higher levels of the benchmark structure.

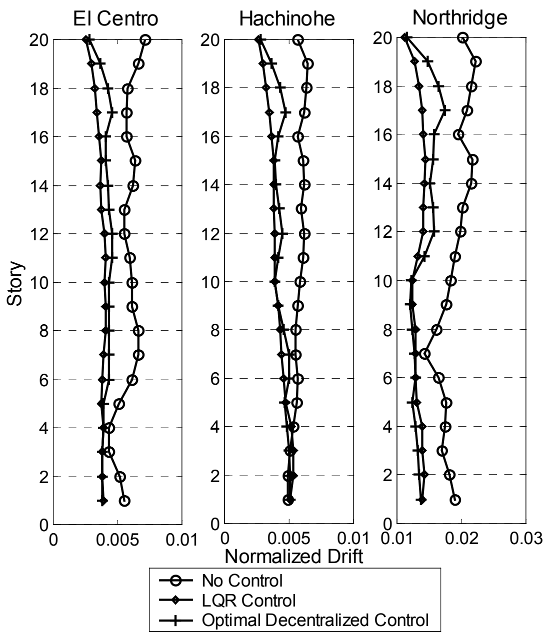

Figure 7 – Inter-story Drift Demands: Centralized LQR versus the Optimal Decentralized Controller

6. MARKET BASED CONTROL

In observing free capitalistic markets, it is evident that through the individual intentions of both consumers and sellers alike, an efficient means of societal resource allocation exists. The complex laws of supply and demand are the fundamental building blocks in determining the equilibrium price of goods in a decentralized economy. Borrowing the concept of a marketplace for application in a control system is rather new to the field of control. Investigations have been made into the use of market-based control techniques in the realm of MEM's (micro-electrical machines), computer networks as well as in computer resource allocation problems [13]. Lynch and Law proposed to implement a market-based control strategy for structural control [14]. The resulting market-based controller is decentralized with respect to knowledge of the system plant. In terms of *a posteriori* information, the approach can be easily decentralized with a hierarchical information control structure during implementation.

A structure employing a large number of sensors and actuators can be modeled as a market place that centers on the buying and selling of the scarce resource of power. The cost efficiency of any structural control system is indirectly proportional to the consumption of power the system requires. Therefore, allowing actuators to enter a market place for purchasing power, the local interaction of the system's actuators can potentially attain an efficient global control system that can adequately control a structure's deflections while at the same time use power in an efficient manner. The derived control law is termed Pareto optimal since no agent can do any better in the marketplace without diminishing the performance of another [15].

First, a demand function is proposed for the system's actuators that will govern the amount of control power purchased. Numerous demand functions can be proposed, but to make illustration of the market-based control concept simple, a linear demand function is proposed [14].

$$P_{DEM} = -\left|f(x,\dot{x})\right| p + \left|g(x,\dot{x})\right| \qquad (12)$$

The scarce commodity of interest in the system is power, *P*. The control force applied to the structure by each control device will be directly proportional to the amount of power purchased. Each actuator will have an opportunity to purchase at the market's equilibrium price, *p*. It is natural to expect that as the price of power is high relative to the actuator's overall wealth, it's inclined to purchase less. For a linear demand function, the slope and the *P*-axis intercept are sufficient to fully characterize the function. It is proposed that the intercept and the slope of the demand function take on the following form.

$$f(x,\dot{x}) = \frac{1}{x+\alpha\dot{x}} \qquad g(x,\dot{x}) = G(x+\alpha\dot{x}) \qquad (13)$$

Figure 8 depicts how the selection of the demand function's intercept and slope influence the growth of demand for large structural responses.

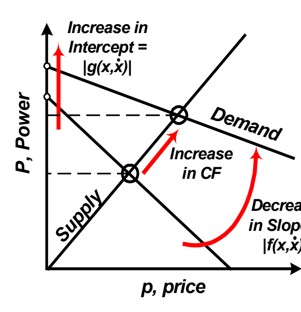

Figure 8 - Growth of Demand and Control Force to Large System Responses

In a similar fashion, a linear supply function is proposed.

$$P = \frac{1}{\beta} p \tag{14}$$

In the implementation phase, supply and demand of the system agents are aggregated and the equilibrium price of power at each time step is determined by finding the point where aggregate supply equals aggregate demand. For the selected supply and demand functions, the resulting control force can be found for the global system. For linear supply and demand functions, the resulting control law is non-linear [14].

Each system agent is provided with an amount of wealth with which it can spend on power purchases. The demand function of each agent is weighted by his respective wealth. If power is purchased by an agent at the market price when $P_{market} < P_{demand}$, then that amount is subtracted from the overall wealth of the agent. .

With appropriate selection of the weighting terms (G, α, and β) used in the system demand and supply functions of each agent, suitable global control can be achieved. Figure 9 depicts the performance of the decentralized market based controller compared to that of the centralized LQR controller. As shown, the performance of the market-based controller can be tuned to have performance characteristics similar to that of the LQR controller.

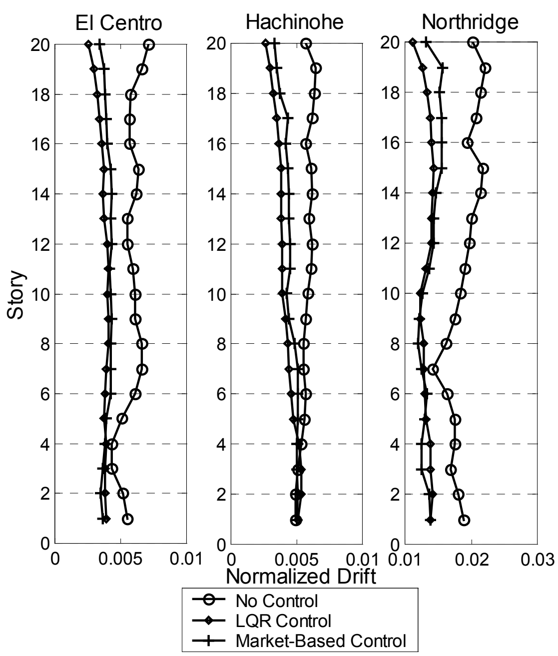

Figure 9 – Inter-story Drift Demands: Centralized LQR versus Market-based Control

7. CONCLUSION

This paper introduced three decentralized control techniques. All three exhibited suitable global performance with the optimal decentralized controller and the market-based controller yielding the best reductions in system response. However, the decentralized extension of the centralized optimal controller represents an easy to implement controller with sub-optimal performance.

Other decentralized control techniques exist and can also be considered for application to the large-scale complex problem of controlling a structure during an earthquake. Decentralized control will become important and better suited in the future for systems employing large numbers of small and inexpensive actuators and sensors.

Given their flexible frameworks, the decentralized control approaches can be extended to handling significantly more complex systems such as nonlinear structures. Many methods of nonlinear structural control depend upon the reduction or the linearization of the system model to a point where classical control approaches can be applied. Alternatively, the complexity of the nonlinear control problem can be best tackled with a distributed control framework with increased complexity being dealt with by each subsystem.

Future investigation will consider the robustness of the decentralized control approaches. An advantage of the decentralization of the controller is robustness in the face of failure. Given one subsystem fails, the other subsystems might be capable of compensating accordingly and ensure suitable global performance of the system. Decentralized control provides a framework to study possible subsystem failures in a distributed control system.

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