EARTHQUAKE ENGINEERING AND STRUCTURAL DYNAMICS
Earthquake Engng Struct. Dyn. 2002; **31**:1855–1877 (DOI: 10.1002/eqe.193)

Market-based control of linear structural systems

Jerome Peter Lynch and Kincho H. Law*,†

Department of Civil and Environmental Engineering, The John A. Blume Earthquake Engineering Center, Stanford University, Stanford, CA 94305, U.S.A.

SUMMARY

To limit the response of structures during external disturbances such as strong winds or large seismic events, structural control systems can be used. In the structural engineering field, attention has been shifted from active control to semi-active control systems. Unlike active control system devices, semi-active devices are compact, have efficient power consumption characteristics and are less expensive. As a result, an environment of a large number of actuators and sensors will result, rendering a complex large-scale dynamic system. Such a system is best controlled by a decentralized approach such as market-based control (MBC). In MBC, the system is modelled as a market place of buyers and sellers that leads to an efficient allocation of control power. The resulting MBC solution is shown to be locally Pareto optimal. This novel control approach is applied to three linear structural systems ranging from a one-storey structure to a 20-storey structure, all controlled by semi-active hydraulic dampers. It is shown that MBC is competitive in the reduction of structural responses during large seismic loadings as compared to the centralized control approach of the linear quadratic regulation controller. Copyright © 2002 John Wiley & Sons, Ltd.

KEY WORDS: market-based control (MBC); semi-active control; active control; decentralized structural control; smart structures; linear quadratic regulation (LQR)

1. INTRODUCTION

To ensure safety and long-term performance, structures are designed to limit their response to various external disturbances such as earthquakes and winds. The current state of practice attains these limits through the use of lateral resisting systems and in some instances with base isolation systems. In the past decade, new and promising control systems have been designed and implemented as an alternative means of limiting structural responses during strong wind and earthquake loads. The control systems designed to date can be broadly

* Correspondence to: Kincho H. Law, The John A. Blume Earthquake Engineering Center, Department of Civil and Environmental Engineering, Stanford University, Stanford, CA 94305, U.S.A.
† E-mail: law@ce.stanford.edu

Contract/grant sponsor: National Science Foundation; contract/grant number: CMS-9988909.
Contract/grant sponsor: Department of Defense's National Science and Engineering Graduate Fellowship.

Received 10 June 2000
Accepted 5 February 2002

classified into two categories: active and semi-active. Active control systems are those that employ large actuators that directly input forces into the structure in an attempt to limit structural deflections. While such systems can be viewed as great achievements, they do suffer from some technological and economic limitations. In response to these limitations, semi-active control systems have emerged. In a semi-active system, devices are used to change the damping or stiffness properties of a structure, thereby indirectly removing energy from the system. Semi-active control systems hold great promise since they can be effectively used during large seismic events, are small in size, consume less power than their active counterparts, and are less expensive to fabricate [1].

A common semi-active control device used to date is the variable hydraulic damper. One example is the semi-active hydraulic damper (SHD), designed by Kajima Corporation, Japan. This damper can modify its damping coefficient in real time through the opening or closing of an orifice valve connecting two adjacent hydraulic chambers. The end result is a control device that can yield a maximum control force of 1000 kN using only 70 W of power. Kurata *et al.* have recently implemented eight SHD dampers in a five-storey structure in Shizuoka, Japan [2]. Other semi-active device mechanisms exist such as controllable fluid dampers and controllable friction dampers. In particular, Spencer *et al.* have designed and tested a controllable magnetorheological fluid damper that controls its damping coefficient by changing a magnetic field surrounding the damper's fluid chamber [3].

If the current trend of evolution continues with semi-active device shape factors shrinking and power consumption characteristics improving, it is likely that structural control systems of the future will employ large numbers of control devices. In the field of structural monitoring, new and innovative low-cost wireless sensors are emerging, as illustrated by Lynch *et al.* and Straser, that will allow control systems to employ more sensors for system state feedback [4, 5]. For such a large-scale complex system, the orthodox approach of using a central computer responsible for the control of the entire system will become less desirable. Another inherent difficulty associated with the widely used centralized controller approach is that one controller represents a single point of failure in the system.

One approach to handling complex control systems is to use a decentralized control system [6]. The attainable benefits of using decentralized control are high system performance in light of system uncertainty, greater stability robustness, improved control system performance in non-linear systems, and system installation modularity facilitating low-cost installations, diagnostics and module replacements [7]. Architecturally, in a centralized control system, one central controller is used to co-ordinate the collection of state information from system sensors and based on these measurements, determine actuation forces for control. In the decentralized system, a central controller will no longer be necessary with control devices housing on-board computational power to facilitate the calculation of their own control actions based on the measurements of the system sensors. Many decentralized control techniques exist that are used in a variety of control systems for non-structural engineering applications. Some decentralized control methods potentially applicable to structural control include decentralized state feedback, decentralized proportional-integral control, and degenerate control, just to name a few [6].

An immediate example of a complex system elegantly controlled in a decentralized fashion is the free market economies. In a free market system, scarce societal resources are distributed based on the local interactions of buyers and sellers who obey the laws of supply and demand as set forth by Adam Smith [8]. In the free markets, what is optimally 'controlled' is the price that is paid for goods and the salary workers receive for their services. The historically poor performance of centrally controlled economies is additional evidence of the difficulty associated with controlling a complex marketplace.

This paper is an investigation into the development of a novel control technique termed market-based control (MBC) for application to structural control systems. In market-based control, the complex dynamic system is modelled as a market whose operation is akin to financial markets. A scarce system resource is identified and is optimally distributed in a decentralized manner. Researchers have investigated the use of MBC techniques to micro-electro-mechanical systems (MEMS) where hundreds of actuators and sensors are employed in system plants of high uncertainty with high likelihood of actuation failure [9]. In the area of computer architecture, market-based control has been applied to problems of optimal resource allocation on computer networks as well as in the time-sharing of microprocessor power for software processes [10]. Market-based control has even been applied to systems that regulate the flow of fluid in tanks [11].

In this paper, the MBC approach is formulated for complex dynamic structural systems. In the formulation, comparisons will be made to the linear quadratic regulation (LQR) control approach that is the widely used centralized control algorithm in practice. A one-degree-of-freedom structure controlled by an actuator will be used for illustration of the effectiveness of market-based control in reducing structural responses during large seismic loadings. The approach will also be applied to larger complex structural systems such as the five-storey Kajima–Shizuoka building as well as a 20-storey steel structure, both using semi-active devices as their primary source of control.

2. LINEAR QUADRATIC REGULATION

In designing a linear control system, the most effective and widely used approach is the centralized LQR. Before beginning a discussion on the derivation of market-based control, the LQR approach is briefly reviewed. Let us first consider a structural system whose equation of motion in state space form is

$$\dot{X}(t) = AX(t) + BU(t) + DW(t) \tag{1}$$

The state of the system, $X(t)$, contains the displacement and velocity response terms of the system. The system is externally loaded by a dynamic disturbance, $W(t)$, and controlled by control forces, $U(t)$. The matrices, B and D, respectively, represent the location of the system actuators and external loads. The eigenvalues of the system matrix, A, characterize the uncontrolled dynamic response of the system. When plotted on the complex plane, these eigenvalues, often termed poles of the system, will all fall in the left half side of the plane if the dynamic system is stable. The right half side of the complex plane represents instability, such that if any system pole is located there, the entire system is dynamically unstable. Graphically, the natural frequency and damping coefficient of each mode of the system can be determined from the location of the poles in the complex plane. The absolute distance from the pole to the origin is the natural frequency of that pole's mode while the sine of the angle between the pole and the positive imaginary axis is the damping ratio of the mode. Figure 1 depicts the graphical relationship between a system pole and its corresponding modal natural frequency and damping ratio.

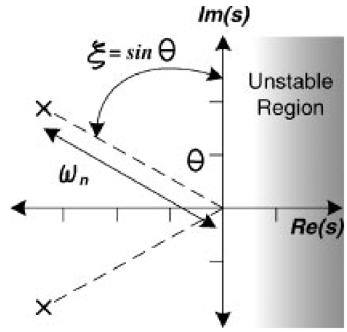


Figure 1. The relationship between system poles and the corresponding modal frequency and damping ratio.

To improve the response of a system subjected to external disturbances, the poles of the system can be moved to more desirable locations on the complex plane. While many pole placement techniques exist, LQR has emerged as a reliable systematic guide to pole placement that allows for the weighting of control response against control effort.

The LQR method provides an optimal control solution through the minimization of a cost function that encapsulates the system's control objectives. The cost function, J, contains two control objectives; the minimization of structural response, $Y(t)$, and the minimization of the input control forces, $U(t)$, required to attain those responses with a weighting matrix, R, which is included to vary the proportional emphasis between the two terms.

$$J = \int_0^\infty (Y^{\mathrm{T}} Y + U^{\mathrm{T}} R U)\,\mathrm{d}t \tag{2}$$

The regulated response vector, $Y(t)$, can be representative of any structural response as long as it can be written as a function of the system state, $X(t)$, with C representing the transformation from the full system state.

$$Y(t) = CX(t) \tag{3}$$

Therefore, we are afforded more flexibility in choosing the desired structural response to be controlled. For example, inter-storey drifts can now be a control objective rather than trying to indirectly control drifts by controlling absolute structural displacements. As a result, the cost function can be written in terms of the state vector where $Q = C^{\mathrm{T}} C$.

$$J = \int_0^\infty (X^{\mathrm{T}} Q X + U^{\mathrm{T}} R U)\,\mathrm{d}t \tag{4}$$

To ensure that a minimum of the cost function can be found, the weighting matrices, Q and R, must both be positive definite. The positive definite criterion guarantees that the surface of the cost function is upward convex. Furthermore, the quadratic form of the cost function is necessary to avoid the minimum point existing as a cusp point and thus unobtainable.

The result of the minimization of the cost function is a static gain matrix, K, that when multiplied by the full state of the system, $X(t)$, yields the optimal control force vector.

$$U(t) = -R^{-1} B^{\mathrm{T}} P X(t) = -KX(t) \tag{5}$$

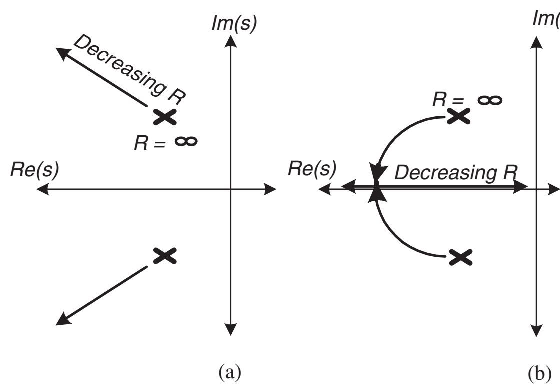


Figure 2. Influence of the system cost function on closed-loop pole locations.

The Ricatti matrix, P, represents the solution of the algebraic Ricatti equation that results in the minimization procedure.

$$PA + A^{\mathrm{T}}P + Q - PBR^{-1}B^{\mathrm{T}}P = 0 \tag{6}$$

Substituting Equation (5) into Equation (1), a revised state equation of the system can now be expressed as

$$\dot{X}(t) = (A - BK)X(t) + DW(t) \tag{7}$$

The new pole locations of the closed-loop system are the eigenvalues of the modified system matrix, $A' = A - BK$. Key to understanding the location of the closed-loop poles is the understanding of the influence of the individual terms of the LQR cost function. The route the migrating poles take from their initial uncontrolled position to their LQR closed-loop locations is dependent on the variable, $Y(t)$, chosen to be regulated [12]. If $Y(t)$ is equal to the vector of displacements of the system nodes, the poles will migrate in a manner consistent to increased system stiffness. Increased system stiffness is synonymous with poles migrating outward as shown in Figure 2(a). On the other hand, if the regulated response is system velocities, the resulting control solution will cause poles to migrate consistent with increased system damping. Poles rotating about the origin towards the negative real axis is consistent with increased system damping, as shown in Figure 2(b). A combination of displacement and velocity in the regulation variable would result in a pole migration pattern that would be influenced by both increased system stiffness and damping. How far the final poles result on these generalized trajectory paths is dependent upon the weighting matrix R. If R is near infinity, the poles will not move since this makes control effort expensive. As R decreases towards 0, control becomes inexpensive and poles result in positions far from their open-loop positions.

As illustrated by Equation (5), the calculation of the control forces for the system requires the full state, $X(t)$, at each time step. In practice, a centralized controller is used to take measurements from the system sensors, assemble the state vector, and calculate the control commands for the control devices.

The limitations of the LQR controller should be noted. In particular, the optimality of the LQR solution is dependent upon the assumption of a linear system. Application in structures excited by large seismic events, the assumption of system linearity is invalid. For use in non-linear systems, the non-linearity of the structure has to be modelled in the analysis and an additional control loop is designed for the control system that cancels the system non-linearities. Another inherent weakness of the LQR method is its heavy dependence upon the assumption of perfect knowledge of the system. If the model used in the design of the controller is incorrect, the LQR solution is no longer optimal and could even be destabilizing.

3. FORMULATION OF MARKET-BASED CONTROL

3.1. Overview

Structures excited by large external disturbances are complex systems characterized by high dimensionality, non-linear behaviour and significant plant uncertainty. Introducing a large number of control devices and measuring system responses from hundreds of sensors, a large information network emerges rendering the control problem even more complex. The control task at hand cannot be solved by simply using faster computers [13]. Rather, new ideas are needed for decomposing the overall system down to independent sub-systems that can be handled locally by decentralized controllers. The control devices in a decentralized system act as independent decision makers in a global system. The goal of the system is to allocate system resources (i.e. input device power) in a distributed and efficient manner.

A strong parallel exists between the operational goal of a decentralized control system and the free market economies. Both seek to attain an efficient distribution of scarce system resources. In the case of an economic market, goods and services serve as the scarce resource while power is the scarce resource in a control system. Using the rules observed in a free market for the operation of control devices in a control system, MBC emerges as a viable control technique. Market-based control is a multi-objective optimal control technique that exploits the efficiencies of a marketplace for determining the allocation of power in a control system.

Control devices and power sources represent agents in the marketplace of power. In particular, control devices would represent the buyers while the power sources used to supply power to the system represent the market sellers. In a decentralized sense, individual sellers seek to maximize their profits while buyers desire to maximize the utility attained through purchasing power. Associated with each seller of power is a cost function, J_S, that is a function of the amount of power produced, P_S, and the price of power, p, at each time step. The seller's cost function is a representation of the amount of profit the seller obtains when selling power at the market price. Likewise, for each buyer there is an associated cost function, J_B, that is a function of the system response of the buyer's subsystem, $Y(t)_j$, the amount of power sought, P_B, and the price of power, p. For the buyer, this cost function is also termed the buyer's utility function since it represents the amount of utility a buyer derives by purchasing an amount of power.

In a decentralized fashion, each individual buyer and seller seeks to maximize their associated cost functions at each time increment of the system.

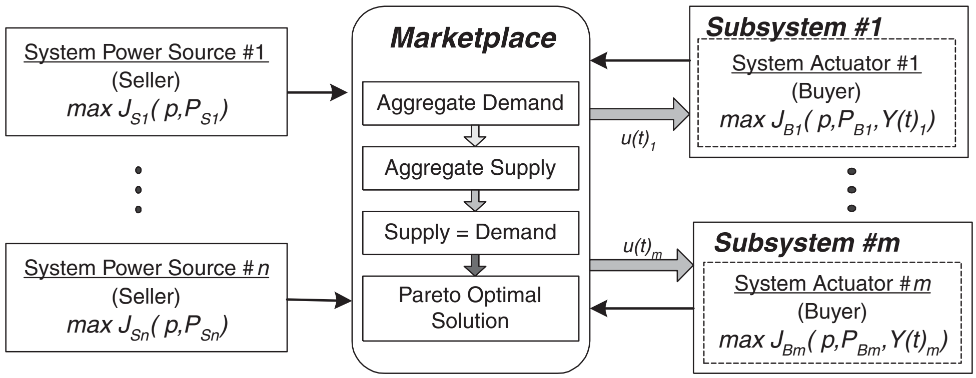


Figure 3. Overview of the MBC system's control objectives.

$$\begin{aligned} &\max J_{S1}(P_{S1}, p)\\ &\max J_{S2}(P_{S2}, p)\\ &\vdots\\ &\max J_{B1}(P_{B1}, Y(t)_1, p)\\ &\max J_{B2}(P_{B2}, Y(t)_2, p)\\ &\vdots \end{aligned} \tag{8}$$

The system is constrained by the conservation of total energy in the system where the m system buyers purchase all the power produced by the system's n suppliers.

$$\sum_{i=1}^{n} P_{Si} = \sum_{j=1}^{m} P_{Bj} \tag{9}$$

Each buyer's utility function is maximized with respect to power to determine the demand function of the buying agent. For the selling agents, the cost function is also maximized with respect to power to derive the seller's supply function. However, for an efficient global solution to result, the maximization process of each agent is constrained by the optimization of all the other ongoing system agents. The marketplace facilitates the simultaneous optimization of all the constraining cost functions of the market agents.

The marketplace aggregates the demand functions of the individual buyers to obtain the demand function of the market. In a likewise manner, the market aggregates the supply functions of the sellers to determine the market's supply function. At each point in time, the demand function and supply function of the market share a point where they intercept. This equilibrium point represents the state of competitive equilibrium of the system that sets the equilibrium price of power. Such a solution is an optimal solution in the multi-objective optimization sense. Economists classically term this optimal condition as Pareto optimal [14]. At this Pareto optimal equilibrium point, each agent in the economy is doing as well as it can given the action of all other system agents. No agent can do better without adversely affecting the benefit of another agent. With the price set, each buyer purchases power that is in turn used to apply control forces to the system. Figure 3 serves as an illustration of the role the marketplace plays in the determination of system control forces, $U(t)$.

3.2. Single-degree-of-freedom derivation

First, a single-degree-of-freedom system is considered in the derivation of the demand and supply functions of the system. Later, these simplified functions are extended with minimal modification to their multiple-degree-of-freedom form.

There do not exist strict rules on the form of the control devices' demand functions. In this study, the demand functions of the market's buyers are designed to be directly influenced by two factors: the price of power and the response of the structure to extreme disturbances. Consistent with consumers in a market, when the price of power is low, system control devices are inclined to purchase more power at the market price. On the other hand, when the price of power rises, devices will purchase less. This implies a decreasing demand function for increasing power cost. The demand function designed will also be directly influenced by structural responses so that when the response of a structure increases, so will the demand of the control devices. While many forms can be chosen (linear, parabolic, exponential, etc.), a linear demand function is proposed for this study because of its simplicity. For a linear demand function, the slope and y-axis intercept of the function are sufficient to fully characterize the function. A negative slope will ensure that for rising costs, power demand will decrease.

$$P_{\text{DEMAND}} = -|f(x,\dot{x})|\,p + |g(x,\dot{x})| \tag{10}$$

The slope and intercept point of a device's demand function, f and g, will vary with the displacement and velocity of the control device's node. An absolute value is taken of the functions relating system response to slope and intercept since these values must be positive in the form shown in Equation (10). When the structure's displacement and velocity increase, it is natural to expect the demand for power to grow proportionally. Allowing the y-axis intercept to increase with increasing structural responses can attain an increase in demand. Likewise, reducing the demand function's slope also attains increased demand. The demand function's slope and y-intercept take the following form:

$$f(x,\dot{x}) = \frac{1}{Tx + Q\dot{x}} \tag{11}$$

$$g(x,\dot{x}) = Rx + S\dot{x} \tag{12}$$

where T, Q, R, and S, represent the various constants used for tuning the market-based controller. Assuming a linear supply function, Figure 4 depicts how the selection of the demand function's intercept and slope influence the growth of the demand function.

When the price of power increases, system sellers will produce more to reap greater revenues. As a result, a linear supply function is selected that increases with an increase in the price of power. When the price of power is zero, no producer can gain revenue and therefore no power is produced. This observation requires the supply function to intercept the origin as depicted in Figure 4. The slope of the supply function is chosen to be the constant $1/\beta$.

$$P_{\text{SUPPLY}} = \frac{1}{\beta}\,p \tag{13}$$

For a single-degree-of-freedom system, the point of competitive equilibrium is where the supply and demand functions intersect. For the selected supply and demand functions, the

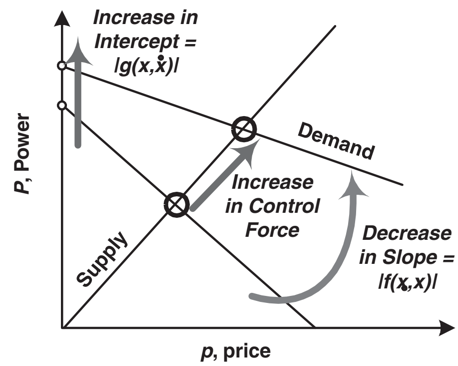


Figure 4. The behaviour of the buying agent's demand function.

resulting equilibrium price of power can be found by equating Equations (10) and (13). Using this equilibrium price, the amount of power purchased by the control device can be determined using Equation (10). The control force, U, is proportional to the amount of power purchased by the proportionality constant, K.

$$U = K\left(\frac{|Rx + S\dot{x}|}{|Tx + Q\dot{x}| + \beta}\,Tx + \frac{|Rx + S\dot{x}|}{|Tx + Q\dot{x}| + \beta}\,Q\dot{x}\right) \tag{14}$$

The resulting control force is quite different in form to that obtained from an LQR analysis. The control law from LQR is linear while for market-based control, the control law of Equation (14) is non-linear. The coefficients of the displacement and velocity terms of the control law for the MBC approach vary in relation to the structural response. This causes a constant movement in the closed-loop system poles. This is in contrast to the fixed pole locations of the LQR control law.

3.3. Market-based control of a single-degree-of-freedom structure

To illustrate the effectiveness of the MBC method, a single-degree-of-freedom system is considered. A linear lumped mass shear model of a one-storey structure is considered subjected to the unscaled El Centro (1940, NS) seismic disturbance. Both an LQR and MBC algorithm are implemented for comparison of control system performance. The structure's natural period is 0.5 s with a mass of 158 000 kg and a stiffness of 25 000 kN/m. The structure's damping ratio is assumed to be 5%. For this illustrative structure, no specific type of control device (active nor semi-active) is specified in order to allow the designed LQR and MBC controllers to implement control forces without concern to device limitations. However, a maximum allowable control force of 1000 kN is imposed on the system actuator.

The uncontrolled response of the structure exhibits a maximum absolute displacement of 4.43cm. First, an LQR controller is designed and implemented. For the design of the controller, the weighting matrix, Q, was selected with a weighting only on the velocity term of the state vector to cause an increase of damping of the system. The weighting term on actuation effort,

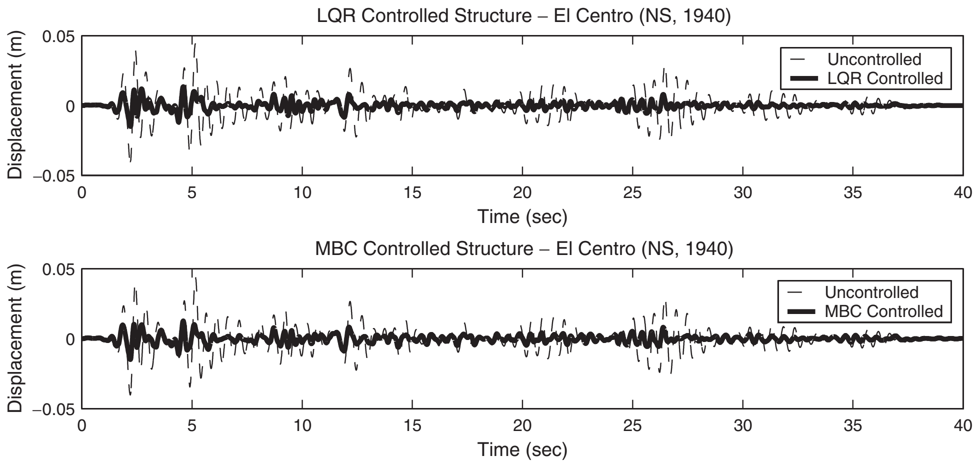


Figure 5. Time history response of controlled SDOF structure due to El Centro (NS, 1940).

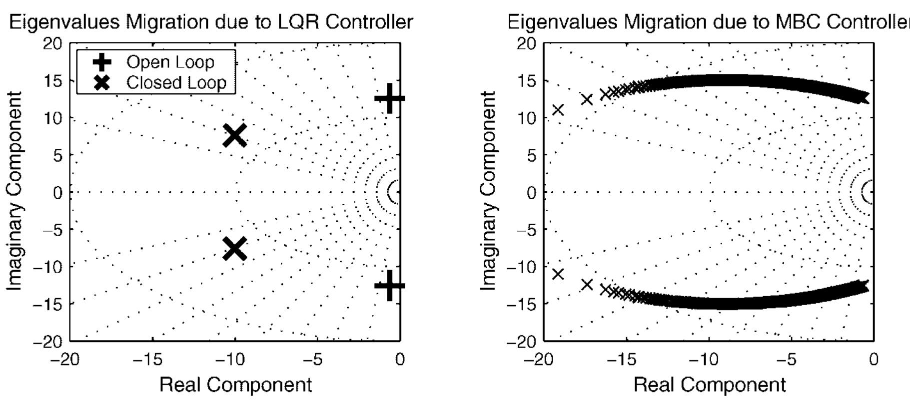


Figure 6. The migration of system eigenvalues due to the LQR and MBC controllers.

R, was varied until the actuator exceeded its capacity. They are given as

$$Q = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad R = 1 \times 10^{-13} \tag{15}$$

The result was the static control gain $G = [-1.1257 \times 10^6,\ 2.8255 \times 10^6]$ that when implemented, reduced the maximum absolute displacement to 1.51 cm. Figure 5 shows the time history response of the controlled structure subjected to the El Centro earthquake. As seen in Figure 6, the result of the LQR controller is the relocation of the system's open-loop eigenvalues to new locations in the left half side of the complex plane. The new locations have increased damping (0.79) with the original natural frequency held constant.

Next, a market-based controller was implemented in the system. The goal of the MBC controller was to attain the same level of response reduction as the LQR controller. The control system designer is free to determine how to select the weighting terms of the market functions. For example, selection of the controller's weighting terms, Q, R, S, T, and β, could be made from a detailed parameter study of the controlled response of the system to various seismic records under a plethora of different weighting terms. However, for this study, the weighting terms are chosen based on engineering judgment of the expected response of the system. To maintain a perspective of how the market mechanism is operating, the weighting terms are selected to normalize the slope and intercept of the demand function at the root mean square values of the uncontrolled system response. This way, large incomprehensible values of power and price do not cloud our visibility of the market's operation. The root mean square value of the uncontrolled response is a convenient normalizing point due to its representation of energy contained in the uncontrolled system.

Considering the response of the uncontrolled structure, the root mean square of the system's uncontrolled displacement and velocity is 0.01 m and 0.12 m/s, respectively. Scaling these values to 1, the scaling factors on displacement and velocity are 100 and 8, respectively. These scaling factors serve as our starting point for the values of T and Q, except that Q is increased to 12 to represent an increased importance placed on system velocities. The slope of the supply function, β, is set to 1. To allow for significant variation of the coefficients of the displacement and velocity terms of Equation (14), R and S are both selected to be 1. With the weighting factors of the market defined, the control weighting term, K, is to be determined. The actual price of power and the amount purchased based upon the supply and demand functions of the system, will both be <1. Therefore, K is varied until desired control results are attained. A scaling factor of 2.2×10^6 is selected for K. The result is a reduction of system displacement to a maximum absolute displacement of 1.50 cm. Figure 5 shows the time history response of the system with an MBC controller. The results for the single-degree-of-freedom system clearly indicate that a controller based on the MBC approach is as effective as an LQR controller when properly designed.

Figure 6 is an illustration of the migration pattern of the system eigenvalues due to the MBC controller. Different from the LQR's static closed loop pole locations, the eigenvalues of the MBC controlled system change with system response measurements. If structural responses are large, the poles migrate further to the left side of the complex plane. The trajectory pattern is consistent with the weighting terms chosen. We select the weight on state response, Q, to weight system velocities more than displacements. The result is a trajectory that is consistent with greater damping during large system responses with a small increase in the natural frequency of the system. If only increased damping was desired as seen in the LQR controller, then the weighting term on displacement, T, should be set to 0.

While the performance of the two systems are quite similar, does one control methodology require more control effort than the other? In consideration of this question, the absolute value of the control force of each controller is added in time. As shown in Figure 7, a plot of the accumulated control effort of both controllers reveal that for the single-degree-of-freedom system, the MBC controller attains the same control performance with 40% less total control effort. The reduced control effort of the market-based controller can be attributed to the adaptive nature of the controller's feedback coefficients shown in Equation (14). When system responses become large, the feedback coefficients of the market-based controller increase, giving greater control effort when needed. This contrasts with the fixed gain of the LQR controller.

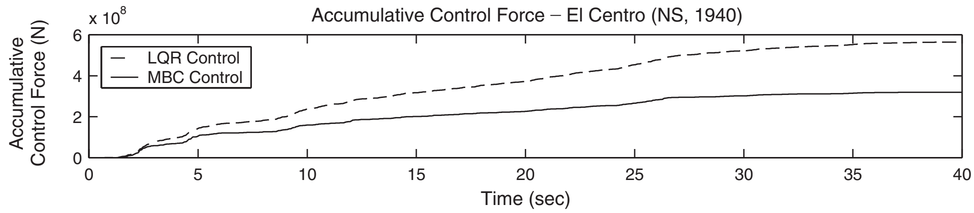


Figure 7. Accumulated control forces of the LQR and MBC controllers.

3.4. *Multi-degree-of-freedom derivation*

While the results from the single-degree-of-freedom system are evidence of the merits of the MBC algorithm, it represents a simplification since the realistic application of MBC is in large-scale systems. In this section, the basic laws of supply and demand developed for the single-degree-of-freedom system are extended to multiple-degree-of-freedom systems.

In large-scale systems of multiple control devices acting in the role of buyers and system power sources representing market sellers, a true marketplace appears. Not considered in the discussion of the single-degree-of-freedom case, but necessary in an environment of multiple buyers and sellers, the concept of wealth is introduced. Each buying agent of the system has an allocation of wealth, W_i, with the subscript i denoting the ith agent of the market. Total wealth in the system, W_{TOTAL}, is maintained constant.

$$W_{\mathrm{TOTAL}} = \sum_{i=1}^{m} W_i \tag{16}$$

The demand function of each agent will be influenced by the amount of wealth the agent possesses. If a buying agent possesses a large amount of saved wealth, then its demand for power would naturally increase, just as affluent consumers spend more than destitute ones in a real economy. To depict this, the demand function used in the single-degree-of-freedom system is augmented with the agent's wealth in the following manner:

$$P_{D_i} = \left(-\left| \frac{1}{Tx_i + Q\dot{x}_i} \right| p + |Rx_i + S\dot{x}_i| \right) W_i \tag{17}$$

For the selling agents of the system, they are not permitted to maintain corporate wealth. Therefore, no modification is necessary of the supply functions of the sellers. As a result, they distribute the money they make back to the market buyers. Each buyer receives an equal share of the total profit made by the market sellers, regardless of the amount of power purchased by that buyer. This is analogous to the pay of corporate workers who are also consumers in the marketplace.

The total market demand at each time step is determined by aggregating the demand functions of all m buying agents of the market. In a similar fashion, the supply functions of the system's n selling agents are aggregated to form a global market supply function. The competitive equilibrium price of power is obtained from the point where the demand of the market equals supply. The result is the equilibrium price of power at each time step.

$$p_{\text{eq}} = \frac{\sum_{i=1}^{m} W_i |Rx_i + S\dot{x}_i|}{n/\beta + \sum_{i=1}^{m} W_i / |Tx_i + Q\dot{x}_i|} \tag{18}$$

Again, T, Q, R, and S, are weighting terms of the system buyers' demand functions while β is the inverse proportionality constant of the sellers' supply function. Once the market price is established, each control device will only buy power if the market price at that time step does not exceed its wealth. Once all actuators have purchased power, the amount paid is subtracted from their total wealth.

3.5. Stability of the control solution

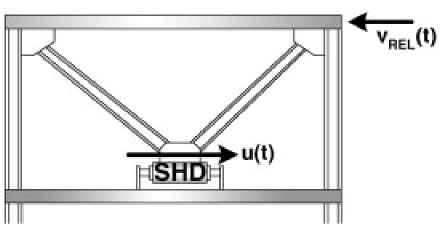
Stability is defined by a system's tendency to grow or decay in response to an input disturbance to the system. If the response decays, the system is considered stable. However, if the response grows in time, then the system is defined as unstable. The stability of a dynamic system is characterized by the location of the system poles in the complex plane. Given the existence of at least one pole in the right half part of the complex plane, the system is considered unstable and will exhibit growing system response to input disturbances. If all poles are located in the left half part of the complex plane, the system is stable. Various tests for linear system stability exists such as the Routh's stability criterion and the Nyquist stability criterion [15].

For the LQR controller, closed-loop system stability is guaranteed if two criteria are met: (a) if the system matrix, A, and the control location matrix, B, of Equation (1) are a controllable pair; and (b) R and Q of Equation (4) are both positive definite [12]. The controllability criterion ensures that the controller has influence on all modes, particularly unstable modes, of the system. The positive definite criteria on R allows for control effort to have a positive effect on the cost function, J, while the positive definite criterion on Q provides penalty on system responses, particularly unstable responses.

Unlike the LQR controller, the current controller derived in the MBC method has not been shown to be mathematically stable in closed form. Further work is needed to consider the limitations of the controller with regard to stability concerns. However, when applied in systems that utilize semi-active control devices, stability is less of a concern since the devices do not add mechanical energy directly to the system and therefore the system is bounded-input bounded-output (BIBO) stable [16].

4. MULTIPLE-DEGREE-OF-FREEDOM ANALYSIS

Market based control is applied to two multiple-degree-of-freedom systems. The first structure considered is the Kajima–Shizuoka Building recently constructed in Shizuoka, Japan [2]. The structure is a five-storey steel structure employing eight semi-active hydraulic dampers (SHD) on the first four stories of the structure. The second system is a hypothetical 20-storey steel structure employing a total of 48 SHD dampers distributed throughout the structure. In simplifying the analysis, each structure is modelled as a lumped mass shear model permitted to sustain elastic deformations. A rigorous analysis of response was not the desired goal of this study, in which case the second order inelastic response of the structural components would



Maximum Control Force	1,000 kN
Maximum Displacement	+/- 6 cm
Stiffness of the SHD	400,000 kN/m
Maximum Damping Coefficient	200,000 kN-s/m
Minimum Damping Coefficient	1,000 kN-s/m
Maximum Shaft Velocity	25 cm/s
Power Consumption	70 Watts

Figure 8. Application of the Kajima SHD.

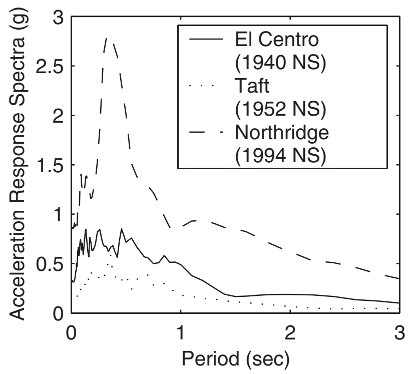


Figure 9. Acceleration response spectra of the seismic disturbances used (damping ratio 5%).

have to be considered. Rather, our concern is in evaluating the effectiveness of the MBC solution within a linear system. Only major reductions of displacements are sought from the uncontrolled to the controlled response of the linear structures utilizing an LQR and MBC controller. However, as will be seen, the responses of both structures utilizing control are within their elastic limits.

In both structural systems, SHD variable dampers are used as the primary control devices. Typically, the SHD damper is installed between the low point of a stiff K-brace and the floor. Given a command control force, the SHD calculates the damping coefficient by dividing the command force by the relative velocity between the two floors to which the SHD is connected. If the relative velocity between the two floors where an SHD is attached is in an opposite direction as the desired control force, then the control force is applied. If the response is in the same direction, no control force is applied and the damper is set to a default minimum value. Figure 8 illustrates the installation of the SHD control device and lists the operational properties of the damper.

Given the flexibility of the K-brace, the SHD damper and the brace are coupled and analysed as one Maxwell damping element [17]. A Maxwell element is a dashpot and spring in series whose force response, $p(t)$, is characterized by a second-order differentiable equation.

$$\dot{p}(t) + \frac{k_{\text{eff}}}{c_{\text{SHD}}}\, p(t) = k_{\text{eff}}\, \dot{x}(t) \tag{19}$$

The combined stiffness of the SHD damper in series with the brace represents the effective stiffness, k_{eff}, of the Maxwell element.

To evaluate the effectiveness of the MBC strategy, two far field earthquake records and one near field earthquake record are used. El Centro (1940 NS) and Taft (1952 NS) represent the far field records while Northridge (1994 NS—Sylmar County Hospital) is the near field record selected. All three earthquakes are scaled so that their maximum absolute velocities are 50 cm/s. The peak ground acceleration of the three scaled earthquake records are 3.07, 1.53, and 8.27 m/s^2, respectively. The acceleration response spectra of the input earthquakes are shown in Figure 9.

5. NUMERICAL SIMULATION OF THE SHIZUOKA FIVE-STOREY STRUCTURE

The Kajima–Shizuoka Building is a five-storey steel structure roughly 19 m in height. With plan dimensions of 11.8 m by 24 m, the structure has significant flexibility in the direction of its short plan dimension [2]. As a result, the designed lateral resisting system for this structure's direction is augmented with a total of eight semi-hydraulic dampers upon the first four floors of the structure. Two SHD dampers are used per floor, all placed at the apex of K-braces. The associated stiffness of the K-braces are chosen to ensure deformation is dominated by the dampers resulting in better performance when the dampers are controlled as compared to using them in a passive capacity. The stiffness of the K-braces on the first floor are 438 kN/mm each while the stiffness of all braces upon the second through fourth storey are 565 kN/mm. Figure 10 shows the structural details of the Kajima–Shizuoka Building as well as the location of all SHD devices. The periods of the five modes of response for the structure are 0.99, 0.35, 0.22, 0.17 and 0.15 s.

The intent of the design engineers was to effectively reduce structural displacements by operating the system's SHD dampers through a centralized control system. Kurata *et al.* has

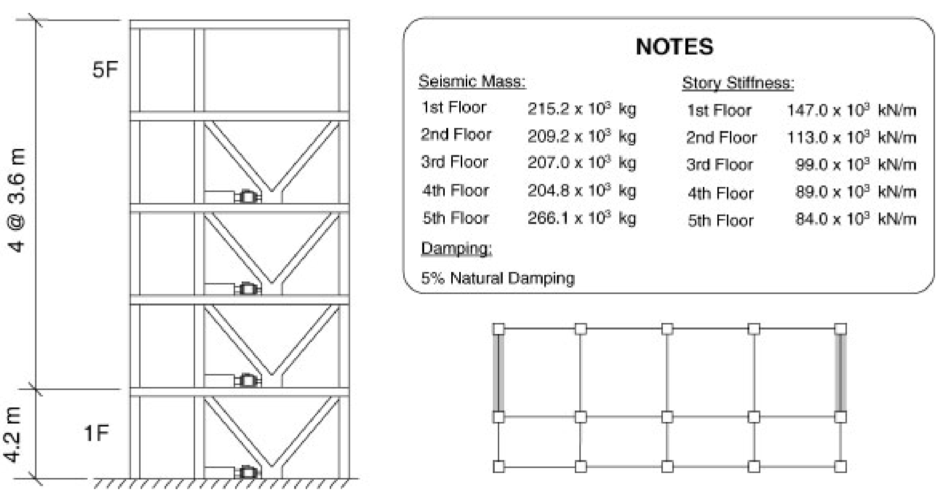


Figure 10. Kajima–Shizuoka Building.

designed and implemented an LQR strategy for the Kajima–Shizuoka Building and has shown its effectiveness in attaining the control objective of reducing inter-storey drifts [2]. In this study, a market-based controller will be designed and implemented. To assess the benefits of the decentralized MBC controller, a centralized LQR controller will also be utilized in controlling the Kajima–Shizuoka building. Both controllers will be designed with respect to their performance during the scaled El Centro seismic disturbance. Once implemented, the control systems are analysed during the scaled Taft and Northridge records.

The design of an MBC controller is considered first. Its design is dependent upon the prudent selection of the weighting coefficients of the supply and demand functions depicted in Equations (13) and (17). Drawing on the success of the selection process used for the single-degree-of-freedom system, the weighting terms of the supply and demand functions are selected in a similar fashion. First, given that the fifth storey displacement and velocity response of the uncontrolled system is greatest, they are considered in the tuning of the weighting terms. Under the scaled El Centro disturbance, the root mean square of the uncontrolled fifth storey displacement and velocity time histories are determined to be 0.07 m and 0.34 m/s. The root mean square values will be used to determine the weighting terms of the demand function's slope, Q and T. To scale the root mean square values to 1, the weighting term on the slope function's displacement and velocity are about 14 and 3, respectively. This will in effect set the slope of the demand function to 1 for the root mean square response of the fifth storey. However, to give more emphasis to the control system's velocity solution versus letting the control system be dependent upon increasing system stiffness, the Q term is increased by three fold to 9. Considering the weighting terms of the demand function's intercept, R and S, they are scaled to give an intercept of 1 for the root mean square values of displacement and stiffness. A weight of 8 is given to R and a weight of 2 to S. The slope of the supply function is set to one. The conversion factor between power and control force, K, is used to tune the system to ideal performance. The following summarizes the MBC controller's weighting terms:

$$R=8;\quad S=2;\quad T=15;\quad Q=10;\quad \beta=1;\quad K=1\times 10^5 \tag{20}$$

Given the dominance of the first mode of response of the system, maximum structural responses under disturbances are assumed to increase with increasing height of the structure. To give buying agents in the upper stories more initial wealth to which to purchase power from the system sellers, the following initial wealth distribution is used:

$$W_1=100;\quad W_2=150;\quad W_3=300;\quad W_4=400;\quad W_5=0 \text{ (no control device)} \tag{21}$$

Roughly speaking, the Pareto optimal price of competitive equilibrium of the system that results will be centered close to 1. In light of this fact, the values of the initial wealth for the buyers are chosen such that the buyers will not deplete their wealth too quickly during a seismic disturbance.

For the design of the LQR controller, the weighting matrix on state response, Q, is selected with the objective of reducing system velocity responses. The weighting on control, R, is increased to a point of actuation saturation. The following summarizes the weighting matrices used to derive the LQR gain matrix, K, of Equation (5):

$$Q=[[0]\quad [I]] \quad \text{and} \quad R=1\times 10^{-13}[I] \tag{22}$$

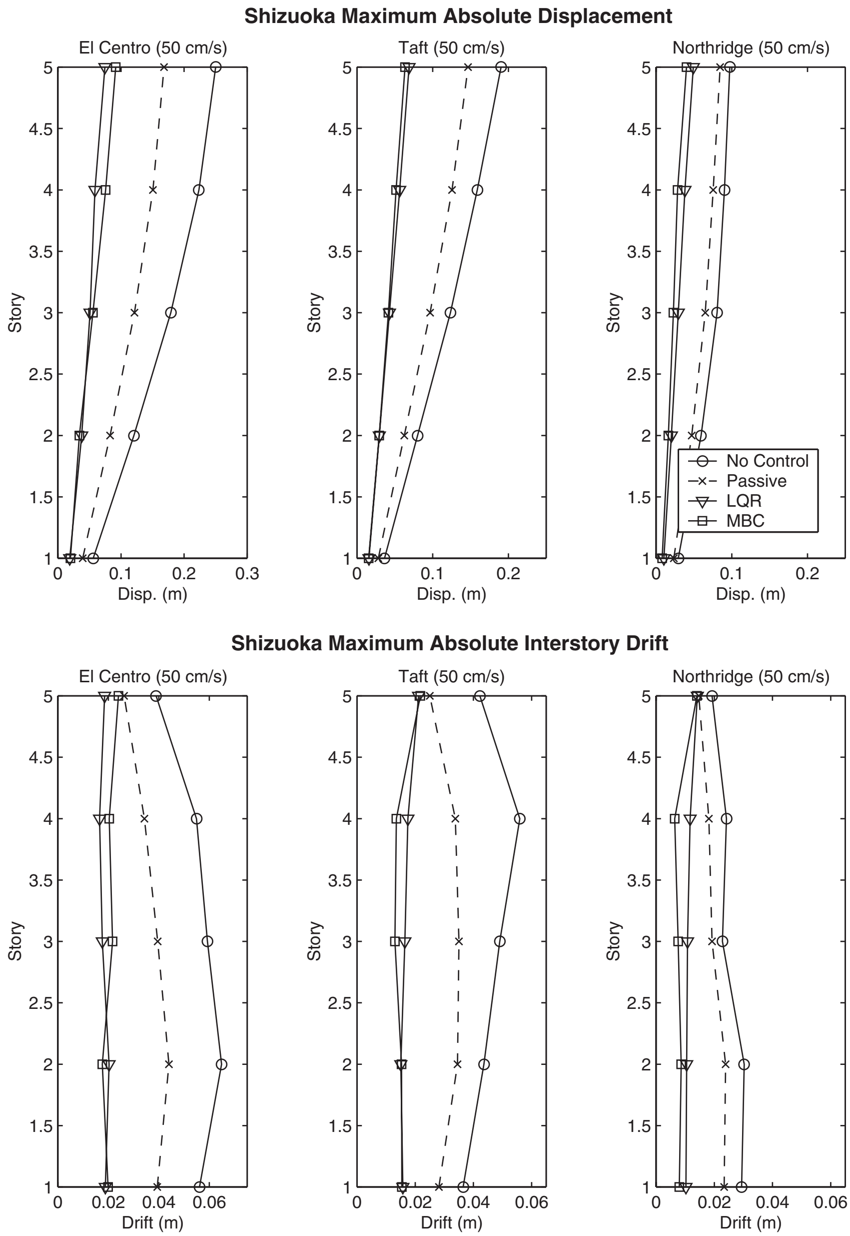


Figure 11. Control applied to the Kajima–Shizuoka Building—LQR versus MBC.

As can be seen in Figure 11, the reduction of both maximum absolute displacements and maximum inter-storey drifts are quite significant for both the LQR and MBC controller when compared to the uncontrolled and passively controlled response. In comparing the performance of the MBC and LQR controllers, it can be safely concluded that both yield similar reductions

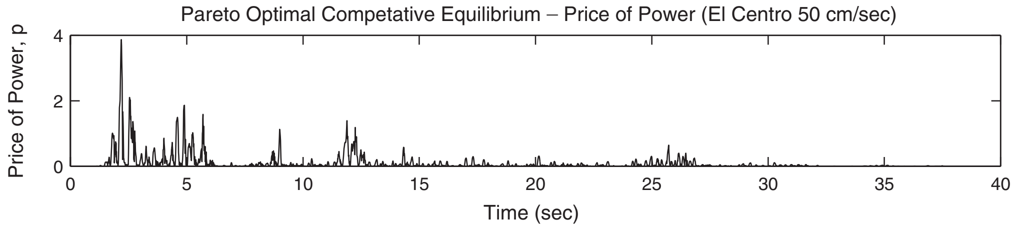


Figure 12. Time history of the Pareto optimal solution of the MBC system.

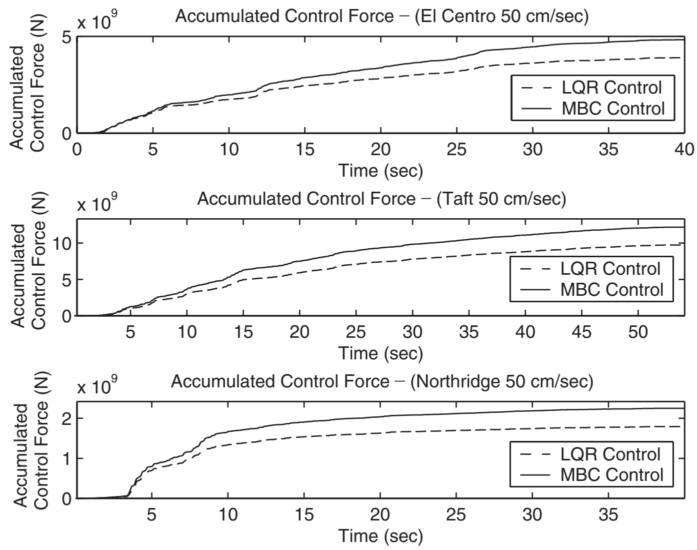


Figure 13. Accumulated control effort of Kajima–Shizuoka LQR and MBC controllers.

of structural response. In the instance of the Taft and Northridge earthquakes, the MBC controller is marginally better while for the El Centro record, the LQR controller is marginally better.

In considering the market mechanism, it is interesting to consider the variation of the Pareto optimal solution with respect to time. To do so, let us consider the variation in the market price of power shown in Figure 12 for the El Centro analysis. Clearly, a strong relationship exists between the equilibrium price of power and the input ground motion to the structure. As the structural responses of the system increase with respect to the input ground acceleration, the demand for control power increases. Given the fixed nature of the supply function, the result is an increase in the equilibrium price of power. Therefore, the behaviour of the Pareto optimal price depicted in Figure 12 is consistent with the expected market response for the input system disturbance.

Again, the total control effort of the control system is considered to ensure that the MBC controller is not using excessive amounts of control energy to remain competitive with the

LQR solution. As seen in Figure 13, the MBC controller is using approximately 25% more control effort during all three seismic disturbances. With a different approach to tuning the MBC market function's coefficients, the control effort of the MBC could be reduced.

6. NUMERICAL SIMULATION OF A 20-STOREY LARGE-SCALE STRUCTURE

To illustrate the effectiveness of the MBC method, a larger structural system is considered. In particular, the 20-storey steel structure depicted in Figure 14 is selected. A total of 48 SHD control devices are distributed through out the structure to resist seismic disturbances in the lateral direction. Four SHD dampers are used upon each of the first six floors, two per floor for the 7th to the 16th floor while the upper four floors only have one variable damper per floor. This distribution should be sufficient in reducing the uncontrolled lateral response of the structure during large-scale earthquakes. Given the size of the structure and the large number of system control devices in use, the associated benefits of pursuing a decentralized control approach makes MBC a prudent choice.

Again, the top storey response is considered in the determination of the weighting coefficients used in the MBC controller design. The root mean square displacement of the

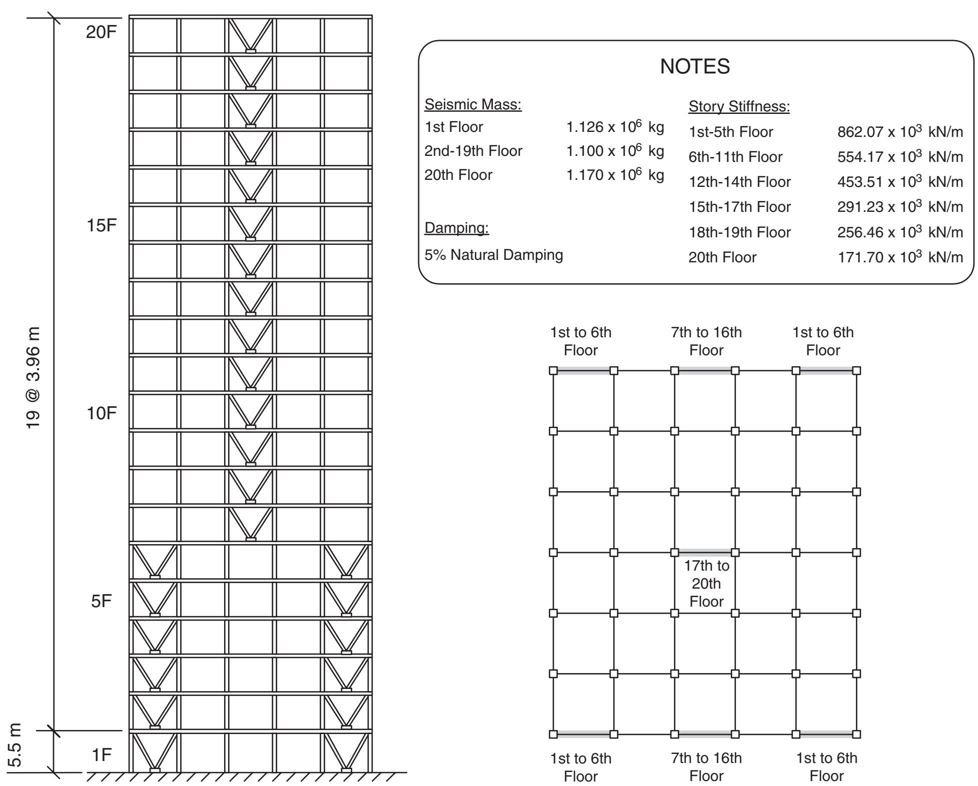


Figure 14. Twenty-Storey steel structure for analysis.

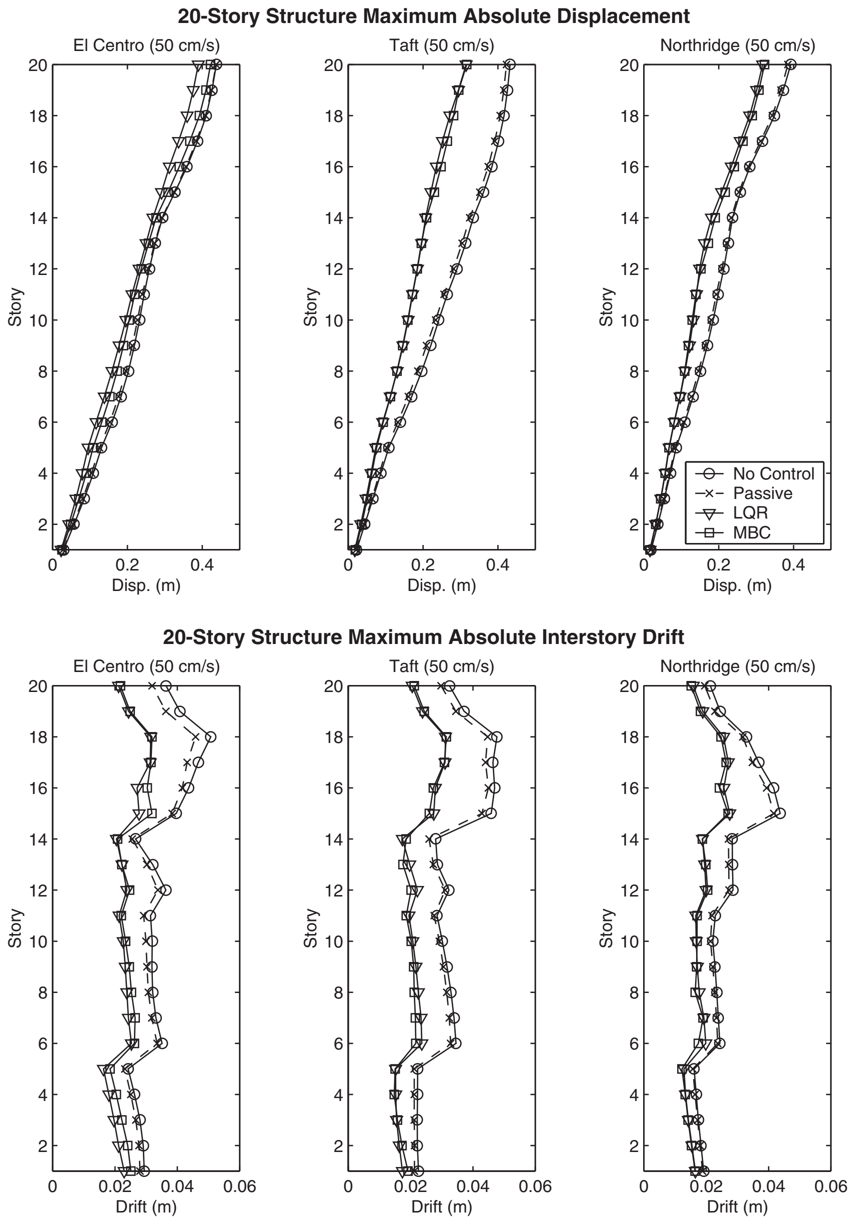


Figure 15. Control applied to the 20-storey structure—LQR versus MBC.

uncontrolled response to the El Centro seismic record is determined to be 0.1353 m while the root mean square velocity of the top storey is 0.3195 m/s. As a result, T and Q, are initially chosen to be 8 and 3, respectively. To emphasize that the controller increases damping instead of stiffness, the Q term is increased to 6. To normalize the intercept of the demand function

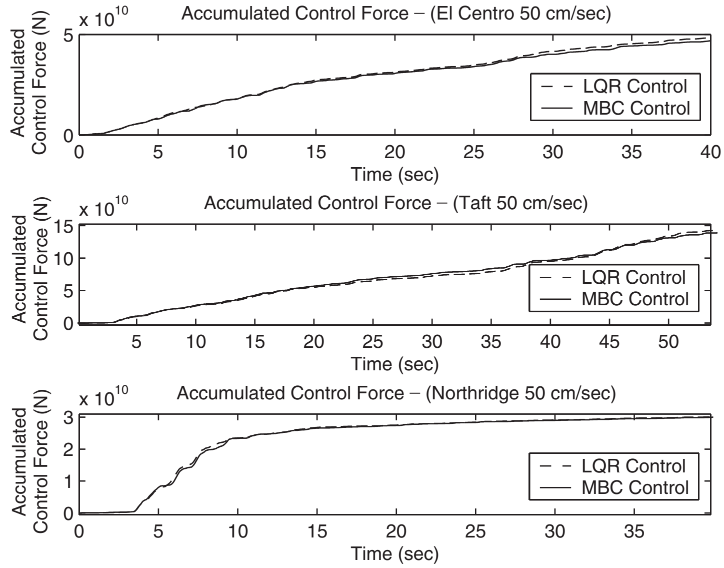


Figure 16. Accumulated control effort of 20-storey structure using LQR and MBC controllers.

to the root mean square values of displacement and velocity, the R and S weights are chosen to be 4.5 and 1.5, respectively. Again, the slope of the supply function is fixed to 1. K is varied until two control goals are attained: desirable control results are obtained and control effort of the MBC controller is within 5% of the LQR controller. The resulting weighting terms are presented in Equation (23).

$$R=4.5;\quad S=1.5;\quad T=8;\quad Q=6;\quad \beta=1;\quad K=2\times10^5 \tag{23}$$

For the design of the LQR controller, the weighting matrices selected are similar to those used in the LQR controller design in the previous Shizuoka building example.

$$Q=[[0]\quad [I]] \quad \text{and} \quad R=1\times10^{-14}[I] \tag{24}$$

Figure 15 depicts the effectiveness of the LQR and MBC controllers. As can be seen from the results, the responses obtained from both controllers are again nearly identical. They are both effective in reducing the response of the structure by as much as 50% for some floors of the system.

Not only has the selected gain, K, attained suitable performance levels, but, as shown in Figure 16, it has also been correctly tuned to keep the MBC control effort within 5% of the LQR control effort. For both the El Centro and Taft seismic records, the LQR controller requires as much as 3% more effort than the MBC controller. For the Northridge disturbance, the control efforts of both controllers are within 1% of each other.

7. CONCLUSION

This paper has presented the formulation and implementation of market-based control in structural systems. It has been shown that the control technique can successfully limit the response of structures during large seismic events. In particular, the MBC solution was applied to two BIBO-stable structures that were controlled by semi-active control devices. In comparison to the widely used LQR centralized controller, the decentralized MBC controller was shown to be just as effective and in some instances marginally better. The success of the MBC controllers in comparison to the LQR controllers is evidence of the Pareto optimal solution the MBC market attains.

The merits of MBC are evident and as a result, MBC represents one potential alternative to the centralized LQR solution typically used in structural control systems. Instances where MBC controllers are potentially applicable are in large-scale complex systems that render themselves difficult to control with a centralized controller. The decentralized MBC method is partially suited for control systems that employ a wireless monitoring system. To date, structural monitoring systems used for control are designed around the architecture of the control system. However, new and innovative wireless monitoring systems are being developed. The low installation costs and high reliability associated with these systems could be a strong catalyst in their adoption. In wireless monitoring systems, a linear relationship exists between the distance information is transmitted and the power used for transmission. The inherent power costs associated with inter-sensor communication makes for easy incorporation into the market-based control pricing structure. As a result, the optimal control law will not only consider actuation power but also sensor power costs. A control technique such as MBC can be designed around the sensing system ushering in a new sensor-centric approach to control system design. Future work will include how MBC performs in comparison to other competing decentralized control techniques which could further provide some insights in improving the MBC approach.

In conclusion, a framework of controlling a structure in a decentralized fashion has been proposed. While the linear demand and supply functions chosen are effective and allow for an easy interpretation of the control problem, they are by no means the only form demand and supply functions can take. Furthermore, the method used in choosing the weighting terms within the demand and supply function can be made more consistent with a physical understanding of the system plant. Extension of the MBC control technique could include a means of adaptive market functions to account for actuation failures, changes in the plant properties, and failure within the monitoring system. For this study, the systems considered were linear in response. Further studies are needed in the application of the MBC technique in highly non-linear structural systems. The application of MBC to non-linear systems is currently being investigated.

ACKNOWLEDGEMENTS

This research is partially supported by the National Science Foundation (NSF) under Grant No. CMS-9988909. We gratefully acknowledge the fruitful suggestions received from Dr Andrew Berlin and Dr Warren Jackson of Xerox, PARC, Narito Kurata, Tomohiko Hatada and Akihiro Kondo of the Kobori Research Complex of Kajima Corporation, and Professor Stephen Rock of Stanford University. The

Department of Defense's National Science and Engineering Graduate Fellowship has provided additional support.

REFERENCES

1. Soong T, Spencer Jr B. Active, semi-active and hybrid control of structures. *Bulletin of the New Zealand National Society for Earthquake Engineering* 2000; **33**:387–402.
2. Kurata N, Kobori T, Takahashi M, Niwa N, Midorikawa H. Actual seismic response controlled building with semi-active damper system. *Earthquake Engineering and Structural Dynamics* 1999; **28**:1427–1447.
3. Spencer Jr B, Yang G, Carlson J, Sain M. Smart dampers for seismic protection of structures: a full scale study. *Proceedings of the 2nd World Conference on Structural Control*, Kyoto, Japan, 1998; 417–426.
4. Lynch J, Law K, Straser E, Kiremidjian A, Kenny T. The development of a wireless modular health monitoring system for civil structures. *Proceedings of the Second MCEER Workshop on Mitigation of Earthquake Disaster by Advanced Technologies*, Las Vegas, NV 2001.
5. Straser E. A modular, wireless damage monitoring system for structures. *Ph.D. Thesis*, Department of Civil and Environmental Engineering, Stanford University, 1998.
6. Siljak D. *Decentralized Control of Complex Systems*. Academic Press: Boston, 1991; 1–65.
7. Lukas M. *Distributed Control Systems*: *Their Evaluation and Design*. Van Nostrand Reinhold Company: New York, 1986; 1–16.
8. Smith A. *An Inquiry into the Nature and Cause of the Wealth of Nations*. Rowman and Littlefield: Maryland, 1992.
9. Guenther O, Hogg T, Huberman B. Controls for unstable structures. *Smart Structures and Materials 1997*: *Mathematics and Control in Smart Structures* SPIE ref. no. 3039, pp. 754–763.
10. Clearwater S. *Market-Based Control*: *A Paradigm for Distributed Resource Allocation*. World Scientific Publishing: Singapore, 1996.
11. Voos H, Litz L. Market-based optimal control: a general introduction. *Proceedings of the American Control Conference*, Chicago, IL, 2000; 3398–3402.
12. Stengel R. *Optimal Control and Estimation*. Dover Publications: New York, 1994; 541–571.
13. Lunze J. *Feedback Control of Large-Scale Systems*. Prentice-Hall: New York, 1992.
14. Mas-Colell A, Whinston M, Green J. *Microeconomic Theory*. Oxford University Press: New York, 1995; 307–349.
15. Franklin G, Powell J, Emami-Naeini A. *Feedback Control of Dynamic Systems*. Addison-Wesley Publishing Company: MA, 1994.
16. Housner G, Bergman L, Caughey T, Chassiakos A, Claus R, Masri S, Skelton R, Soong T, Spencer B, Yao J. Structural control: past, present, and future. *Journal of Engineering Mechanics* (ASCE) 1997; **123**:927.
17. Hatada T, Kobori T, Ishida M, Niwa N. Dynamic analysis of structures with Maxwell model. *Earthquake Engineering and Structural Dynamics* 2000; **29**:159–176.